Asymmetric information allocation to avoid disastrous outcomes

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Abstract

We consider an optimal organization structure in terms of information allocation. In a unique implementation problem of desirable effort levels in the context of team production as in Holmstrom (1982) and Winter (2004), we find an important channel through which information structure affects implementation cost. Under certain conditions, this channel makes it optimal to asymmetrically inform the agents, even if they are ex ante symmetric.

KEYWORDS: Moral hazard, Unique nash implementation, Asymmetric information.

JEL Classification: D21, D23, D86

1 Introduction

In an organization, agents are often allocated different tasks, resources, and information. To better understand desirable organization design, it is important to investigate why agents should sometimes be treated asymmetrically.

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Among the many situations in which an organization exhibits asymmetric treatment of agents,¹ this paper analyzes information allocation. Specifically, we ask why agents are sometimes asymmetrically informed. Of course, if agents engage in different tasks or have different characteristics, there would be trivial situations where an asymmetric information structure arises naturally. Therefore, to examine if there is any "intrinsic" reason to allocate information asymmetrically between the agents, we consider a situation where the agents are otherwise completely symmetric, in terms of their characteristics and the tasks they engage in. In more general environments where agents are in fact asymmetric in these aspects, our analysis should be interpreted as examining one of the key (intrinsic) motivations for information allocation.

A related question is why many organizations in reality are not fully transparent. For example, in a company, some information is often kept within a subset of managers even if the information is relevant to the entire company. In chain stores, managers of company-owned stores are usually exposed to more information through periodic meetings than are franchisees. To explain such phenomena, it is important to understand key channels through which information structure affects organization performance.

In this paper, we establish a novel, nontrivial channel between information allocation and implementation cost, in the context of team production à la Holmstrom (1982). We show that, under certain conditions, this channel makes it optimal to have an asymmetric information structure among the agents, even though they are ex ante symmetric. We interpret this channel as a source of an intrinsic motivation for asymmetric information allocation in an organization. As comparative statics, we also examine how the optimal organization structure in terms of information allocation would vary depending on the nature of the environment, such as the impact of information on production technology.

¹The existing studies consider favoritism in organizations (Prendergast and Topel, 1993, 1996)), internal politics (Milgrom and Roberts, 1986, 1990; Milgrom, 1988), strategic uncertainty (Winter, 2006, 2010) and so on.

To be more precise, we consider the following team production model based on Holmstrom (1982).² Two agents engage in a single project choosing binary efforts. The principal, a residual claimant, offers a bonus contract contingent on the binary outcome of the project. The success probability of the project depends on the agents' total efforts, and a binary state of the world. Information structure simply refers to how many agents could observe the realization of the state.

We examine the optimal information structure and optimal bonus contract in this environment, when the principal's goal is to implement the agents' high effort given every realization of the state.³

We answer this question in two sub-cases: First, we consider implementation of high effort in *one of the Bayesian equilibria*, as in the standard approach in the literature. We show that informing no agent is optimal: if an agent is informed, the bonus must be sufficiently high to incentivize high effort for every state, while if s/he is uninformed, incentivizing high effort for an average state would be enough. Thus, making every agent uninformed dominates, regardless of the nature of the problem.

However, implementation in one of the equilibria is often criticized because it implicitly assumes that the agents play the best equilibrium for the principal even if there are other equilibria. Hence, we believe it is important to examine an alternative scenario where the optimal contract and information structure implements the agents' high effort in *every* Bayesian equilibrium (or equivalently, making the agents' high effort the *unique* Bayesian equilibrium), as studied by Winter (2004) without state uncertainty. This unique implementation approach would particularly be relevant, for example, if the failure of the project is extremely hazardous to the principal (e.g.,

²We make a number of simplifying assumptions to highlight information allocation, but we believe that similar insights should hold in more general environments.

³For the analysis of optimal information structure, there is no loss of generality in focusing on implementation of high effort given every realization of the state, because otherwise the problem is trivial: if the principal wants to make an agent to change his action contingent on the state, the agent *must be* informed.

accidents in a plant, loss of a brand's reputation, and so on). Or more generally, when a contract and organization must be designed much in advance of the actual agents' effort choice, it may be difficult for the principal to foresee which equilibrium may be played. The literature also discusses possibilities of collusion among the agents, especially if other equilibria are more preferable to the agents than the best one for the principal. In these situations, it would be reasonable to imagine that the principal may desire to guarantee the agents' high effort not only in the best equilibrium, but also in the other equilibria.

For the unique implementation problem, it can be beneficial for the principal to inform just one of the two agents, hence asymmetric information allocation. To provide rough intuition, assume a convex success probability function as in Winter (2004) so that the best-equilibrium implementation does not imply a unique implementation. Note first that, if both are uninformed, then the bonus should make an agent choose the high effort even if the other chooses the low effort (otherwise, both choosing low would be an equilibrium) given the average state. Informing an agent would have two effects. As a direct effect, the informed agent's incentive would depend on the state, and thus, making him choose the high effort in the good state becomes easier even if the other chooses the low effort. This has the indirect effect that the uninformed agent's incentive becomes easier to satisfy because his partner works hard, at least with a positive probability. Given this, in the bad state, it is enough to incentivize the informed agent *given the* uninformed agent works hard. Therefore, such an asymmetric information structure would be better than no information if "incentivizing the informed agent in the bad state given the uninformed works" is easier than "incentivizing an uninformed agent in the average state given the other shirks". As suggested in this argument, the optimal information structure would vary depending on parameters of the model such as the distribution of the state variable, production technology, and so on. We perform comparative statics in more detail later.

The multiple equilibrium problem in a moral hazard model with many agents is pointed out in Mookherjee (1984). Ma (1988) shows that a mechanism with communication solves the problem without any additional cost if agents' outputs are individually measured. In team production à la Holmstrom (1982), Arya, Glover, and Hughes (1997) introduces a mechanism in which agents report an expected outcome from production and have an option to quit, and argue that such mechanism achieves the implementation cost arbitrarily close to second-best. Winter (2004) consider an optimal mechanism without communication in a team production model and show that it is optimal to pay a different reward for each agent even if agents are completely symmetric. In line with Winter (2004), our paper does not consider a mechanism with communication. Instead, our paper analayzes an effect of asymmetric information structure on the multiple equilibrium problem, which is not examined in Winter (2004).

The paper is structured as follows. Section 2 introduces the model, and Section 3 studies the optimal bonus contract and information structure in a simple two-agent case with symmetric contracts. Section 4 generalizes the result to N-agent cases. Section 5 shows that the symmetric contract assumption can be weakened without changing our qualitative results, although the analysis becomes more complicated. Section 6 concludes.

2 Model

We develop a team production model with one manager (a principal) and n workers (agents) who engage in a project. Each worker $i \in \{1, \ldots, n\}$ simultaneously chooses an effort level $e_i \in \{0, 1\}$, which costs ce_i for c > 0. The profit of the project is $y \in \{S, F\}$ (S > F) and $p_{\theta}(x)$ denotes the probability of a success (y = S), which depends on the agents' joint effort $x = \sum_i e_i$ and task environment $\theta \in \{H, L\}$. We assume that $p_{\theta}(x)$ is increasing for any θ . $\theta = H$ is generated with probability f and $\theta = L$ with 1 - f. For notational convenience, $p_{\phi}(x) = fp_H(x) + (1 - f)p_L(x)$.

The marginal productivity of effort, which is denoted by $\Delta p_{\theta}(x) \equiv p_{\theta}(x) - p_{\theta}(x-1)$, has the property that (i) $\Delta p_H(x) \geq \Delta p_L(x)$ for all x = 1, 2, and (ii) $\Delta p_{\theta}(x) \geq \Delta p_{\theta}(x-1)$ for all θ . (ii) means that collaboration between the agents is essential for the project.⁴

Information structure We compare three kinds of information structures:

- No information (NI): no agent observes θ ,
- Dispersed information (DI): agent $i \in \{1, ..., m\}$ does not observe θ but agent $j \in \{m + 1, ..., n\}$ observes,
- Full information (FI): all agents observe θ .

To represent the above information structure, we introduce the following information set,

$$\Theta_i = \begin{cases} \{H, L\} & \text{if agent } i \text{ observes } \theta, \\ \{\phi\} & \text{if agent } i \text{ does not observe } \theta, \end{cases}$$

where $\phi = \{(H, L)\}$. A strategy profile is denoted by $e = (e_1, \ldots, e_n) = (\{e_1(\theta)\}_{\theta \in \Theta_1}, \ldots, \{e_n(\theta)\}_{\theta \in \Theta_n})$. We call a strategy profile full effort if $e_i(\theta) = 1$ for all i and θ . Throughout this paper, we assume that the principal cannot observe the task information.

Contract A bonus contract is a pair (b, w) such that w is paid if y = F, and b + w is paid if y = S. Note that (i) a contract can not be contingent on the agents' message and (ii) a contract is anonymous.⁵ Furthermore, the agents are protected by limited liability constraints, i.e.,

$$w \ge 0$$
 and $w + b \ge 0$ for all *i*. (LLC)

⁴If $\Delta p_{\theta}(x) < \Delta p_{\theta}(x-1)$ for all θ , a multiple equilibria problem described later does not arise. See Winter (2004).

 $^{{}^{5}}$ We relax the anonymous contract assumption in section 5.

Incentive-inducing contract We define $B_s(e)$ as a set of the bonus contract in which e is a Nash equilibrium under the given information structure $s \in \{NI, DI, FI\}$. We say that the contract b is incentive-inducing (INI) for e^* under information structure s if

$$b \in B_s(e^*),\tag{IC}$$

and

$$b \in \bigcup_{e \neq e^*} B_s(e). \tag{U}$$

The former condition is that e^* is a Nash equilibrium and the latter means that all strategy profiles except for e^* are not a Nash equilibrium.

The principal's optimization problem is written as follows:

$$\sup_{\substack{(w,b,e)\\ \text{s.t.}}} F + p_{\phi}(\sum_{i} e_{i})((S-F) - 2b) - 2w$$

s.t. (LLC), (IC) and (U).

By standard arguments, we observe that w = 0 is optimal. Furthermore, we assume that S - F is sufficiently large to show that full effort is optimal.

Benchmark Before proceeding to analyzing an optimal INI contract, we confirm that dispersed information is never optimal under the standard contract concept in which a full effort profile is one of equilibria.

Depending on the information structure, the set of bonuses such that a full effort profile is a Nash equilibrium, is written as

$$B_{NI}(1,\ldots,1) = \left[\frac{c}{\Delta p_{\phi}(n)},\infty\right),$$

$$B_{FI}((1,1),\ldots,(1,1)) = \left[\max_{\theta \in \{H,L\}} \left\{\frac{c}{\Delta p_{\theta}(n)}\right\},\infty\right),$$

$$B_{DI}(1,\ldots,1,(1,1),\ldots,(1,1)) = \left[\max_{\theta \in \{H,L,\phi\}} \left\{\frac{c}{\Delta p_{\theta}(n)}\right\},\infty\right).$$

An optimal bonus in each information structure is a lower bound of each of the above sets. Dispersed information is never optimal, because in the dispersed information situation the principal has to give an incentive to both ignorant and informed agents.

3 Basic Analysis

To see a benefit of dispersed information clearly, we first consider n = 2 and m = 1.

3.1 No information

We establish the following lemma.

Lemma 1. $B_{NI}(1,0) = B_{NI}(0,1) = \emptyset$.

Proof. To induce $e_i = 1$ and $e_j = 0$, the principal pays $b \ge \frac{c}{\Delta p_{\phi}(1)}$ to agent i and $b \le \frac{c}{\Delta p_{\phi}(2)}$ to agent j. Because $\Delta p_{\phi}(2) \ge \Delta p_{\phi}(1)$, there is no b satisfying $\frac{c}{\Delta p_{\phi}(2)} \ge b \ge \frac{c}{\Delta p_{\phi}(1)}$.

As shown in the lemma, there is no equilibrium with asymmetric effort choices given an anonymous contract. Thus, we can focus on the problem of preventing e = (0,0) from being an equilibrium and making e = (1,1) the unique equilibrium. The next result shows that this problem of eliminating e = (0,0) is the binding constraint in the optimal contract. Our problem is, therefore, essentially different from the standard approach in the literature that aims that the desirable effort pair be one of the possible equilibria.

Proposition 1. Suppose the no-information case. The optimal bonus is $b_{NI} = \frac{c}{\Delta p_{\phi}(1)}$. The implementation cost is given as $\frac{p_{\phi}(2)c}{\Delta p_{\phi}(1)}$.

Proof. By lemma 1, (U) is rewritten as

$$b \notin \bigcup_{(e_1, e_2) \neq (1, 1)} B_{NI}(e_1, e_2) = B_{NI}(0, 0) = \left(-\infty, \frac{c}{\Delta p_{\phi}(1)}\right],$$

$$\leftrightarrow \ b \in \left(\frac{c}{\Delta p_{\phi}(1)}, \infty\right).$$

Therefore, (U) and (IC) are replaced by

$$b \in \left(\frac{c}{\Delta p_{\phi}(1)}, \infty\right) \cap \left(\frac{c}{\Delta p_{\phi}(2)}, \infty\right) = \left(\frac{c}{\Delta p_{\phi}(1)}, \infty\right),$$

e $\Delta p_{\phi}(2) \ge \Delta p_{\phi}(1).$

because $\Delta p_{\phi}(2) \geq \Delta p_{\phi}(1)$.

The crucial assumption of this result is the convexity of the success probability function, p. The convexity of p means that an agent's incentive of choosing the high effort is lower if the other agent chooses the low effort. To see the importance of this condition, consider the bonus level $b = \frac{c}{\Delta p_{\phi}(2)}$. Given this bonus level, e = (1, 1) is an equilibrium, because if agent *i* chooses $e_i = 1$, then it is optimal for $j \neq i$ to choose $e_j = 1$ as well. However, if i chooses $e_i = 0$, then it is optimal for j to choose $e_j = 0$, and thus, there is another equilibrium e = (0, 0). In order to avoid this undesirable equilibrium, we need to set $b = \frac{c}{\Delta p_{\phi}(1)}$ so that, even if the other agent chooses e = 0(or e = 1), it is optimal for an agent to choose $e = 1.6^{67}$

3.2**Dispersed** information

We establish the following lemma.

1. $B_{DI}(1, (0, 0)) = B_{DI}(0, (1, 1)) = \emptyset$. Lemma 2.

- 2. $B_{DI}(1, (0, 1)) = B_{DI}(0, (0, 1)) = \emptyset$.
- 3. $B_{DI}(0,(1,0)) \neq \emptyset$ if and only if $f \leq \frac{\Delta p_H(1) \Delta p_L(1)}{\Delta p_H(2) \Delta p_L(1)}$
- 4. $B_{DI}(1, (1, 0)) \neq \emptyset$ if and only if $f \geq \frac{\Delta p_L(2) \Delta p_L(1)}{\Delta p_H(2) \Delta p_L(1)}$

⁶This relationship between multiple equilibria and the convexity of the production function is in Winter (2004).

⁷To be rigorous, in order for e = (0,0) not to be an equilibrium, we need to set $b > \frac{c}{\Delta p_{\phi}(1)}$, rather than $b = \frac{c}{\Delta p_{\phi}(1)}$. For this openness issue, we follow Winter (2004) by defining the optimal bonus level as the infimum of those that uniquely implement the desired effort level.

Proof. 1. $B_{DI}(1, (0, 0)) = B_{DI}(0, (1, 1)) = \emptyset$

To induce $(e_1, e_2) = (1, (0, 0))$, a contract should satisfy

$$\frac{c}{\Delta p_{\phi}(2)} \ge b \ge \max\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(1)}\} = \frac{c}{\Delta p_L(1)}$$

However, there exists no b satisfying the above condition because $\Delta p_{\phi}(2) > \Delta p_L(1)$.

To induce $(e_1, e_2) = (0, (1, 1))$, a contract should satisfy

$$\frac{c}{\Delta p_H(2)} = \min\{\frac{c}{\Delta p_H(2)}, \frac{c}{\Delta p_L(2)}\} \ge b \ge \frac{c}{\Delta p_\phi(1)}$$

However, there exists no b satisfying the above condition because $\Delta p_H(2) > \Delta p_{\phi}(1)$.

2. $B_{DI}(1, (0, 1)) = B_{DI}(0, (0, 1)) = \emptyset$ To induce $(e_2(H), e_2(L)) = (0, 1)$, a contract should satisfy

$$\frac{c}{\Delta p_H(e_1+1)} \ge b \ge \frac{c}{\Delta p_L(e_1+1)}.$$

However, there exists no b satisfying the above condition because $\Delta p_H(e_1+1) > \Delta p_L(e_1+1)$.

3. $B_{DI}(0, (1, 0)) \neq \emptyset$ if and only if $f \leq \frac{\Delta p_H(1) - \Delta p_L(1)}{\Delta p_H(2) - \Delta p_L(1)}$. Note that

$$B_{DI}(0,(1,0)) = \left\{ b \mid \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)} \ge b \ge \frac{c}{\Delta p_H(1)} \right\}.$$

Then $B_{DI}(0, (1, 0)) \neq \emptyset$ if and only if $\frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)} \geq \frac{c}{\Delta p_H(1)}$. By rearranging this condition, we obtain statement 3 in the proposition.

4. $B_{DI}(0, (1, 0)) \neq \emptyset$ if and only if $f \leq \frac{\Delta p_H(1) - \Delta p_L(1)}{\Delta p_H(2) - \Delta p_L(1)}$. Note that

$$B_{DI}(1,(1,0)) = \left\{ b \mid \frac{c}{\Delta p_L(2)} \ge b \ge \frac{c}{f \Delta p_H(2) + (1-f) \Delta p_L(1)} \right\}.$$

Then $B_{DI}(1, (1, 0)) \neq \emptyset$ if and only if $\frac{c}{\Delta p_L(2)} \geq \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}$. By rearranging this condition, we obtain statement 3 in the proposition.

The first statement says that asymmetric effort choices cannot be equilibria under an anonymous contract. The second statement says that it is impossible in any equilibrium for an agent to play a high effort in the low state, while playing a low effort in the high state. This results from our definition of the states that $\Delta p_H(x) > \Delta p_L(x)$. The last two statements are about the equilibria where the informed agent's action varies with the state. Depending on the parameter values, such "state-responsive" equilibria could arise.

Therefore, there are potentially three types of equilibria to avoid in order to uniquely implement the desirable effort choices e = (1, (1, 1)), as in the following proposition.

Proposition 2. Suppose the dispersed information case. The optimal bonus is $b_{DI} = \max\left\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}, \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}\right\}$. The implementation cost is given as $2p_{\phi}(2)b_{DI}$.

Proof. We prove here that

$$\cup_{(e_1,e_2)\neq(1,(1,1))}B_s(e_1,e_2) = \left(-\infty,\bar{b}\right],\,$$

where $\bar{b} \equiv \max\left\{\frac{c}{\Delta p_H(1)}, \frac{c}{\Delta p_L(2)}, \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}\right\}$. If this statement is true, we obtain the proposition, because (U) and (IC) are rewritten as $b \in (\bar{b}, \infty)$.

By lemma 2, we observe

$$\cup_{(e_1,e_2)\neq(1,(1,1))} B_{DI}(e_1,e_2) = B_{DI}(1,(1,0)) \cup B_{DI}(0,(1,0)) \cup B_{DI}(0,(0,0)),$$

where

$$B_{DI}(1, (1, 0)) = \left\{ b \mid \frac{c}{\Delta p_L(2)} \ge b \ge \frac{c}{f \Delta p_H(2) + (1 - f) \Delta p_L(1)} \right\},\$$

$$B_{DI}(0, (1, 0)) = \left\{ b \mid \frac{c}{f \Delta p_H(2) + (1 - f) \Delta p_L(1)} \ge b \ge \frac{c}{\Delta p_H(1)} \right\},\$$

$$B_{DI}(0, (0, 0)) = \left\{ b \mid \frac{c}{\Delta p_H(1)} \ge b \right\}.$$

We consider three cases; (i) $\bar{b} = \frac{c}{\Delta p_H(1)}$, (ii) $\bar{b} = \frac{c}{\Delta p_L(2)}$, and (iii) $\bar{b} = \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$.

Case (i) Because $\bar{b} = \frac{c}{\Delta p_H(1)}$, $B_{DI}(0, (1, 0)) \subset B_{DI}(0, (0, 0))$ and $B_{DI}(1, (1, 0)) \subset B_{DI}(0, (0, 0))$.

Case (ii) Because $\bar{b} = \frac{c}{\Delta p_L(2)}$ implies that $f \geq \frac{\Delta p_L(2) - \Delta p_L(1)}{\Delta p_H(2) - \Delta p_L(1)}$, $B_{DI}(1, (1, 0)) \neq \emptyset$ (lemma 2). Therefore,

$$B_{DI}(1,(1,0)) \cup B_{DI}(0,(0,0)) = \left(-\infty, \frac{c}{\Delta p_L(2)}\right].$$

Furthermore, $\bar{b} = \frac{c}{\Delta p_L(2)}$ also implies that $B_{DI}(0, (1, 0)) \subset \left(-\infty, \frac{c}{\Delta p_L(2)}\right]$.

Case (iii) Because $\bar{b} = \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}$ implies that $f \leq \frac{\Delta p_H(1) - \Delta p_L(1)}{\Delta p_H(2) - \Delta p_L(1)}$, $B_{DI}(0, (1, 0)) \neq \emptyset$ (lemma 2). Therefore,

$$B_{DI}(0,(1,0)) \cup B_{DI}(0,(0,0)) = \left(-\infty, \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}\right].$$

Furthermore, $\bar{b} = \frac{c}{\Delta p_L(2)}$ implies $B_{DI}(1, (1, 0)) \subset \left(-\infty, \frac{c}{f \Delta p_H(2) + (1-f) \Delta p_L(1)}\right]$.

Each of the terms, $\frac{c}{\Delta p_H(1)}$, $\frac{c}{\Delta p_L(2)}$, and $\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$ corresponds to the infimum bonus level above which the corresponding effort choices, (0, (0, 0)), (1, (1, 0)), and (0, (1, 0)), respectively, would not be an equilibrium. Recall, however, that (1, (1, 0)) and (0, (1, 0)) can be equilibria only for certain parameter values. In this sense, b_{DI} is an upper bound of the optimal bonus, but it may not be tight. Nevertheless, the proposition states that b_{DI} is indeed the tight upper bound (and hence is the optimal bonus). To show this, we observe in the proof that, for parameter values where $\frac{c}{\Delta p_L(2)}$ is the highest of the three terms, $B_{DI}(1, (1, 0))$ is nonempty, and similarly, for parameter values where $\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$ is the highest among the three, $B_{DI}(0, (1, 0))$ is nonempty. **Example** Suppose $p_{\theta}(x) = \beta e^{\alpha x + \theta}$. Then, $\Delta p_{\theta}(x) = \beta e^{\alpha (x-1) + \theta} (e-1)$. With some manipulation, we obtain the following conditions

$$\frac{c}{\Delta p_H(1)} \ge \frac{c}{\Delta p_L(2)} \qquad \text{iff } \alpha \ge H - L,$$

$$\frac{c}{\Delta p_H(1)} \ge \frac{c}{f\Delta p_H(2) + (1 - f)\Delta p_L(1)} \qquad \text{iff } f \ge \frac{e^{H - L} - 1}{e^{\alpha + H - L} - 1},$$

$$\frac{c}{\Delta p_L(2)} \ge \frac{c}{f\Delta p_H(2) + (1 - f)\Delta p_L(1)} \qquad \text{iff } f \ge \frac{e^{\alpha} - 1}{e^{\alpha + H - L} - 1}.$$

By using the above conditions, we show the optimal bonus in Figure 1.



Figure 1: Optimal Contract with Dispersed Information ($\alpha = 1/2$)

Consider, first, the case where $\frac{c}{\Delta p_H(1)}$ is higher than $\frac{c}{\Delta p_L(2)}$ and $\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$. This is likely to be the case when f is large and H - L is small. As shown in the previous proposition, b_{DI} is given by the bonus level above which (0, (0, 0)) is not an equilibrium. To provide intuition, let $b \geq \frac{c}{\Delta p_H(1)}$. Then, the informed agent is better off choosing e = 1 at state H, even if the uninformed agent chooses e = 0. Thus, (0, (0, 0)) is not an equilibrium. (0, (1, 0))is not an equilibrium either, because $b \geq \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$, the uninformed agent now has an incentive to choose e = 1. This is because, when f is large enough, the uninformed agent believes that the informed chooses e = 1 with a high probability. Finally, (1, (1, 0)) is not an equilibrium, because $b \ge \frac{c}{\Delta p_L(2)}$, the uninformed agent has an incentive to choose e = 1 even in state L, if the uninformed chooses e = 1 (in any state). To explain this in more detail, recall that the bonus level is set so that, in state H, the informed is better off choosing e = 1 given the uninformed chooses e = 0. Now consider the informed in state L, but assume that the uninformed chooses e = 1. When H - L is small enough, s/he has a higher incentive to choose e = 1 compared to the case under H and $e_1 = 0$, because the increase in his/her incentive by changing $e_1 = 0$ to 1 outweighs its decrease by changing $\theta = H$ to L.

Next, consider the case where $\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$ is the highest. Compared to the previous case, this is likely to be the case when f is small (and H-L is in a moderate level). To provide intuition, as in the previous paragraph, let $b \geq \frac{c}{\Delta p_H(1)}$ to eliminate a candidate equilibrium (0, (0, 0)). It is not sufficient to uniquely implement (1, (1, 1)), because $\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)} > \frac{c}{\Delta p_H(1)}$. Unless we set $b \geq \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$, (0, (1, 0)) becomes an equilibrium. This is because, when f is sufficiently small, the uninformed agent believes that it is likely that the informed will choose $e_2 = 0$, because the state is likely to be L.

Finally, consider the case where $\frac{c}{\Delta p_L(2)}$ is the highest. This is likely to be the case when H-L is large (and f is in a moderate level). As in the previous two paragraphs, let $b \geq \max\{\frac{c}{\Delta p_H(1)}, \frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}\}$ to eliminate two candidate equilibria (0, (0, 0)) and (0, (1, 0)). However, it is not sufficient to uniquely implement (1, (1, 1)). Unless we set $b \geq \frac{c}{\Delta p_L(2)}$, (1, (1, 0)) becomes an equilibrium. This is because, when H - L is sufficiently large, even if the uninformed agent chooses $e_1 = 1$ in any state, the state being low significantly affects the incentive of the informed agent.

3.3 Comparison

Proposition 3. Full information is never optimal.

Proof. We show that the cost of uniquely implementing e = (1, 1) for any θ under full revelation is always weakly higher than that under no revelation.

Suppose that, given b, e = (1, 1) is a unique Nash equilibrium under full revelation. First, for $\theta = L$, for each i = 1, 2,

$$b \ge \frac{c}{\Delta p_L(2)},$$

so that e = (1, 1) is one of the equilibria. This implies that e = (1, 1) is an equilibrium with no information, because

$$\frac{c}{\Delta p_L(2)} \ge \frac{c}{\Delta p_\phi(2)},$$

and hence $b \geq \frac{c}{\Delta p_{\phi}(2)}$.

Second, because e = (0, 0) is not an equilibrium under full information,

$$b > \frac{c}{\Delta p_L(1)}.$$

This inequality implies that e = (0,0) is not an equilibrium under no information because $\frac{c}{\Delta p_L(1)} \geq \frac{c}{\Delta p_{\phi}(1)}$. e = (0,1) and e = (1,0) are not an equilibrium from lemma 1.

Proposition 4. Dispersed information is better than no information if and only if

$$\frac{\Delta p_L(2) - \Delta p_L(1)}{\Delta p_H(1) - \Delta p_L(1)} \ge f.$$

Proof. Since the implementation cost is represented by $2p_{\phi}(2)b$, we compare b_{FI} and b_{DI} .

If $b_{DI} = \frac{c}{\Delta p_L(2)}$ or $b_{DI} = \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}$, no information is never optimal because

$$\Delta p_{\phi}(1) - \Delta p_{H}(1) = (1 - f)[\Delta p_{L}(1) - \Delta p_{H}(1)] < 0,$$

$$\Delta p_{\phi}(1) - [f \Delta p_{H}(2) + (1 - f) \Delta p_{L}(1)] = f[\Delta p_{H}(1) - \Delta p_{H}(2)] < 0,$$

which respectively implies $b_{FI} > \frac{c}{\Delta p_L(2)}$ and $b_{FI} > \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}$.

Suppose $b_{DI} = \frac{c}{\Delta p_L(2)}$. By rearranging $b_{FI} \ge \frac{c}{\Delta p_L(2)}$, we obtain

$$\frac{\Delta p_L(2) - \Delta p_L(1)}{\Delta p_H(1) - \Delta p_L(1)} \ge f.$$

An important observation in this proposition is that the no-information scenario can be better than the dispersed information scenario, only if $b_{DI} = \frac{c}{\Delta p_L(2)}$. To provide intuition, recall that, in the no-information scenario, the optimal bonus $b_{NI} = \frac{c}{\Delta p_H(1)}$ is exactly the level above which (0,0) is not an equilibrium. In the dispersed information scenario, the corresponding effort profile, (0, (0, 0)), can be eliminated more easily because it is enough to have $b \geq \frac{c}{\Delta p_H(1)}$. This is because the informed agent has more incentive in state H than in the no-information scenario. Hence, $\frac{c}{\Delta p_H(1)} < b_{NI}$. A similar idea applies for another effort profile, (0, (1, 0)). Because the informed agent chooses $e_2 = 1$ in state H, the uninformed agent believes a positive probability for $e_2 = 1$, which makes agent 1 more willing to choose $e_1 = 1$, compared to the no-information scenario where agent 1 believes a zero probability for $e_2 = 1$. Hence, $\frac{c}{\Delta p_H(2) + (1-f)\Delta p_L(1)} < b_{NI}$. Therefore, unless $\frac{c}{\Delta p_L(2)}$ is higher than both $\frac{c}{\Delta p_H(1)}$ and $\frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}$, dispersed information is always better than the no-information scenario.

When $\frac{c}{\Delta p_L(2)}$ is sufficiently high, it is possible that b_{DI} becomes higher than b_{NI} . As discussed before, $\frac{c}{\Delta p_L(2)}$ is the bonus level above which (1, (1, 0))is not an equilibrium. In order to eliminate (1, (1, 0)), we must incentivize the informed agent to choose $e_2 = 1$ in state L, when the uninformed agent chooses $e_1 = 1$. Which is higher depends on the parameter values. On the one hand, it can be more costly than to incentivize an uninformed agent to choose $e_i = 1$ when the other agent chooses $e_j = 0$, because of the state effect: because $\Delta p_L(x)$ is smaller than $\Delta p_{\phi}(x)$, it is more difficult to give an incentive to the informed in state L. On the other hand, it can be less costly, because of the opponent's effort effect: given the effort profile (1, (1, 0)), the uninformed agent chooses $e_1 = 1$. Naturally, when the opponent's effort effect dominates the state effect as in the statement of the proposition, dispersed information is better, and vice versa.

The next example shows this comparison more clearly.

Example Suppose $p_{\theta}(x) = \beta e^{\alpha x + \theta}$. Then, $\Delta p_{\theta}(x) = \beta e^{\alpha (x-1) + \theta} (e-1)$. With some manipulation, we obtain the following conditions

$$\frac{\Delta p_L(2) - \Delta p_L(1)}{\Delta p_H(1) - \Delta p_L(1)} \ge f \text{ iff } \frac{e^{\alpha} - 1}{e^{H-L} - 1} \ge f$$

By using the above conditions, we draw Figure 1.



Figure 2: Dispersed Info. versus No Info. ($\alpha = 1/2$)

4 *n* agents

This section considers the model where a team consists of n agents. Because full information is never optimal, we investigate the other cases.

4.1 No information

Lemma 3. All equilibria are symmetric with respect to agents $(e_1 = e_2 = \cdots = e_n)$.

Proof. Suppose that $e_i \neq e_j$ for some $i, j \in \{1, ..., n\}$. Without loss of generality, we assume $1 = e_i \neq e_j = 0$. Then

$$\frac{c}{\Delta p_{\theta}(x+1)} \ge b \ge \frac{c}{\Delta p_{\theta}(x)},$$

where $x = \sum_{k=1}^{n} e_k$. However, there is no *b* satisfying the above condition because $\Delta p_{\theta}(x+1) \ge \Delta p_{\theta}(x)$.

By the anonymity of contracts, there is no equilibrium with asymmetric effort choices. Thus, $(1, \dots, 1)$ is uniquely implementable at the bonus level in which $(0, \dots, 0)$ is not an equilibrium, which implies that the optimal bonus level is $\frac{c}{\Delta p_{\phi}(1)}$, as in the following proposition.

Proposition 5. Suppose no information. $b = \frac{c}{\Delta p_{\phi}(1)}$ and the implementation cost is $\frac{np_{\phi}(n)c}{\Delta p_{\phi}(1)}$.

Proof. By lemma 3, (U) is rewritten as

$$b \notin \bigcup_{(e_1,\dots,e_n)\neq(1,\dots,1)} B_{NI}(e_1,\dots,e_n) = B_{NI}(0,\dots,0) = \left(-\infty,\frac{c}{\Delta p_{\phi}(1)}\right],$$

$$\leftrightarrow \ b \in \left(\frac{c}{\Delta p_{\phi}(1)},\infty\right).$$

Therefore, (U) and (IC) are replaced by

$$b \in \left(\frac{c}{\Delta p_{\phi}(1)}, \infty\right) \cap \left(\frac{c}{\Delta p_{\phi}(n)}, \infty\right) = \left(\frac{c}{\Delta p_{\phi}(1)}, \infty\right),$$

because $\Delta p_{\phi}(n) \geq \Delta p_{\phi}(1)$.

-	-	-	-	

_

$j \setminus i$	1	0
	$b \ge \frac{c}{\Delta p_{\phi}(n)}$	$b \le \frac{c}{\Delta p_{\phi}(n-m+1)}$
(1,1)	$b \ge \frac{c}{\Delta p_H(n)}$	$b \ge \frac{c}{\Delta p_H(n-m)}$
	$b \ge \frac{c}{\Delta p_L(n)}$	$b \ge \frac{c}{\Delta p_L(n-m)}$
(1, 0)	$b \ge \frac{c}{f\Delta p_H(n) + (1-f)\Delta p_L(m)}$	$b \le \frac{c}{f\Delta p_H(n-m+1) + (1-f)\Delta p_L(1)}$
	$b \ge \frac{c}{\Delta p_H(n)}$	$b \ge \frac{c}{\Delta p_H(n-m)}$
	$b \le \frac{c}{\Delta p_L(m+1)}$	$b \le \frac{c}{\Delta p_L(1)}$
(0,1)	$b \ge \frac{c}{f\Delta p_H(m) + (1-f)\Delta p_L(n)}$	$b \le \frac{c}{f\Delta p_H(1) + (1-f)\Delta p_L(n-m+1)}$
	$b \le \frac{c}{\Delta p_H(m+1)}$	$b \le \frac{c}{\Delta p_H(1)}$
	$b \ge \frac{c}{\Delta p_L(n)}$	$b \ge \frac{c}{\Delta p_L(n-m)}$
	$b \ge \frac{c}{\Delta p_{\phi}(m)}$	$b \le \frac{c}{\Delta p_{\phi}(1)}$
(0,0)	$b \leq \frac{c}{\Delta p_H(m+1)}$	$b \leq \frac{c}{\Delta p_H(1)}$
	$b \le \frac{c}{\Delta p_L(m+1)}$	$b \le \frac{c}{\Delta p_L(1)}$

4.2 Dispersed information

 $i \in \{1, \ldots, m\}$ agent does not know θ and $j \in \{m + 1, \ldots, n\}$ agent does. Assume $n \ge 2$ and $m \ge 1$.

Lemma 4. On every equilibrium, $e_1 = \cdots = e_m$ and $e_{m+1} = \cdots = e_n$.

Proof. Suppose that $e_i \neq e_j$ for some $i, j \in \{1, \ldots, m\}$. Without loss of generality, we assume $(e_i, e_j) = (1, 0)$. Then

$$\frac{c}{\Delta p_{\theta}(x+1)} \ge b \ge \frac{c}{\Delta p_{\theta}(x)},$$

where $x = \sum_{k=1}^{m} e_k + \sum_{k=m+1}^{n} e_k(\theta)$. However, there is no *b* satisfying the above condition because $\Delta p_{\theta}(x+1) \geq \Delta p_{\theta}(x)$. It is similarly shown that $e_{m+1} = \cdots = e_n$.

Therefore, we define

 $B_{DI}(e_i, e_j) = \{b \mid e_1 = \dots = e_m = e_i \text{ and } e_{m+1} = \dots = e_n = e_j \text{ is a NE} \}.$

Lemma 5. 1. $B_{DI}(0, (1, 1)) = B_{DI}(1, (0, 0)) = \emptyset$

- 2. $B_{DI}(1, (0, 1)) \subset B_{DI}(0, (0, 0))$ and $B_{DI}(0, (0, 1)) \subset B_{DI}(0, (0, 0))$.
- 3. $B_{DI}(0,(1,0)) \neq \emptyset$ if and only if $\frac{\Delta p_H(n-m) \Delta p_L(1)}{\Delta p_H(n-m+1) \Delta p_L(1)} \ge f$.
- 4. $B_{DI}(1, (1, 0)) \neq \emptyset$ if and only if $f \geq \frac{\Delta p_L(m+1) \Delta p_L(m)}{\Delta p_H(n) \Delta p_L(m)}$

Proof. 1. $B_{DI}(0, (1, 1)) = B_{DI}(1, (0, 0)) = \emptyset$

- $(e_i, e_j) = (1, (0, 1))$ if $\frac{c}{\Delta p_\phi(n-m+1)} \ge b \ge \frac{c}{\Delta p_L(n-m)}$. But $\Delta p_\phi(n-m+1) \ge \Delta p_L(n-m)$.
- $(e_i, e_j) = (1, (0, 0))$ if $\frac{c}{\Delta p_H(m+1)} \ge b \ge \frac{c}{\Delta p_\phi(m)}$. But $\Delta p_H(m+1) \ge \Delta p_\phi(m)$.
- 2. $B_{DI}(1, (0, 1)) \subset B_{DI}(0, (0, 0))$ and $B_{DI}(0, (0, 1)) \subset B_{DI}(0, (0, 0))$. $B_{DI}(1, (0, 1)) \subset B_{DI}(0, (0, 0))$ because

$$\frac{c}{\Delta p_H(1)} \ge \frac{c}{\Delta p_H(m+1)}.$$

 $B_{DI}(0,(0,1)) \subset B_{DI}(0,(0,0))$ because

$$\frac{c}{\Delta p_H(1)} \ge \min\{\frac{c}{f\Delta p_H(1) + (1-f)\Delta p_L(n-m+1)}, \frac{c}{\Delta p_H(1)}\}.$$

3.
$$B_{DI}(0, (1, 0)) \neq \emptyset$$
 if and only if $\frac{\Delta p_H(n-m) - \Delta p_L(1)}{\Delta p_H(n-m+1) - \Delta p_L(1)} \ge f$.
4. $B_{DI}(1, (1, 0)) \neq \emptyset$ if and only if $f \ge \frac{\Delta p_L(m+1) - \Delta p_L(m)}{\Delta p_H(n) - \Delta p_L(m)}$.

As opposed to the two-agent case, we may have equilibria where informed agents play low effort in state H and high effort in state L (i.e., those of the form (0, (0, 1)) or (1, (0, 1))). If only one agent is informed (as in the dispersed information scenario with two agents), then such an equilibrium does not exist. With multiple informed agents, however, potentially such equilibria may exist if H - L is small enough. Nevertheless, as in the second statement of the proposition, those candidate equilibria of the form (0, (0, 1))or (1, (0, 1)) can be ignored when we investigate the optimal bonus, because the second statement says that the bonus scheme above which (0, (0, 0)) is not an equilibrium, neither (0, (0, 1)) nor (1, (0, 1)) is an equilibrium.

Therefore, to characterize the optimal bonus, it is sufficient to eliminate the three candidate equilibria, (0, (0, 0)), (0, (1, 0)), and (1, (1, 0)). The next proposition is a generalization of Proposition ?? in terms of the numbers of informed and uninformed agents.

Proposition 6. Suppose the dispersed information case. The optimal bonus is $b_{DI} = \max\{\frac{c}{\Delta p_L(m+1)}, \frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}, \frac{c}{\Delta p_H(1)}\}$. The implementation cost is given as $np_{\phi}(n)b_{DS}$.

Proof. In this proof, we prove that

$$\cup_{(e_i, e_j) \neq (1, (1, 1))} B_s(e_1, e_2) = \left(-\infty, \hat{b}\right]$$

where $\hat{b} = \max\{\frac{c}{\Delta p_L(m+1)}, \frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}, \frac{c}{\Delta p_H(1)}\}$. If this statement is true, we obtain the proposition, because (U) and (IC) are rewritten as $b \in (\hat{b}, \infty)$ (note that \hat{b} is larger than the upper bound of $B_{DI}(1, (1, 1))$). By lemma 5,

$$\bigcup_{(e_i,e_j)\neq(1,(1,1))} B_{DI}(e_1,e_2) = B_{DI}(1,(1,0)) \cup B_{DI}(0,(1,0)) \cup B_{DI}(0,(0,0)),$$

where

$$B_{DI}(1, (1, 0)) = \left\{ b \mid \frac{c}{\Delta p_L(m+1)} \ge b \ge \frac{c}{f\Delta p_H(n) + (1-f)\Delta p_L(m)} \right\},\$$

$$B_{DI}(0, (1, 0)) = \left\{ b \mid \frac{c}{f\Delta p_H(n-m+1) + (1-f)\Delta p_L(1)} \ge b \ge \frac{c}{\Delta p_H(n-m)} \right\},\$$

$$B_{DI}(0, (0, 0)) = \left\{ b \mid \frac{c}{\Delta p_H(1)} \ge b \right\}.$$

(i) First, suppose that $\hat{b} = \frac{c}{\Delta p_L(m+1)}$. In this case, we have $B(1, (1, 0)) \neq \emptyset$, because

$$\frac{c}{\Delta p_L(m+1)} \geq \frac{c}{f\Delta p_H(n-m+1) + (1-f)\Delta p_L(1)}$$
$$\geq \frac{c}{f\Delta p_H(n) + (1-f)\Delta p_L(m)}.$$

If
$$\frac{c}{\Delta p_H(1)} > \frac{c}{f\Delta p_H(n) + (1-f)\Delta p_L(m)}$$
, then
 $B_{DI}(1, (1, 0)) \cup B_{DI}(0, (1, 0)) \cup B_{DI}(0, (0, 0)) = B_{DI}(1, (1, 0)) \cup B_{DI}(0, (0, 0))$
 $= (-\infty, \hat{b}).$

If $\frac{c}{\Delta p_H(1)} \leq \frac{c}{f\Delta p_H(n) + (1-f)\Delta p_L(m)}$, then, because $\frac{c}{\Delta p_H(1)} \geq \frac{c}{\Delta p_H(n-m)}$ and $\frac{c}{f\Delta p_H(n-m+1) + (1-f)\Delta p_L(1)} \geq \frac{c}{f\Delta p_H(n) + (1-f)\Delta p_L(m)}$, we have

$$B_{DI}(1,(1,0)) \cup B_{DI}(0,(1,0)) \cup B_{DI}(0,(0,0)) = (-\infty,\hat{b}).$$

(ii) Next, suppose that $\hat{b} = \frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}$. In this case, we have $B(0, (1, 0)) \neq \emptyset$, because

$$\frac{c}{f\Delta p_H(n-m+1) + (1-f)\Delta p_L(1)} \geq \frac{c}{\Delta p_H(1)}$$
$$\geq \frac{c}{\Delta p_H(n-m)}$$

Moreover,

$$B_{DI}(1,(1,0)) \cup B_{DI}(0,(1,0)) \cup B_{DI}(0,(0,0)) = B_{DI}(0,(1,0)) \cup B_{DI}(0,(0,0))$$

= $(-\infty, \hat{b}).$

Each of the terms, $\frac{c}{\Delta p_H(1)}$, $\frac{c}{\Delta p_L(2)}$, and $\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$ corresponds to the infimum bonus level above which the corresponding effort choices, (0, (0, 0)), (1, (1, 0)), and (0, (1, 0)), respectively, would not be an equilibrium. As for the two-agent case, (1, (1, 0)) and (0, (1, 0)) can be equilibria only for certain parameter values, and in this sense, b_{DI} is an upper bound of the optimal bonus. However, the proposition states that b_{DI} is indeed tight. To show this, we observe in the proof that, for parameter values where $\frac{c}{\Delta p_L(m+1)}$ is the highest among the three terms, $B_{DI}(1, (1, 0))$ is nonempty, and similarly, for parameter values where $\frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}$ is the highest among the three, $B_{DI}(0, (1, 0))$ is nonempty. **Example** Suppose $p_{\theta}(x) = \beta e^{\alpha x + \theta}$. Then, $\Delta p_{\theta}(x) = \beta e^{\alpha(x-1)+\theta}(e-1)$. By some manipulation, we obtain the following conditions

$$\frac{c}{\Delta p_H(1)} \ge \frac{c}{\Delta p_L(m+1)} \qquad \text{iff } \alpha m \ge H - L,$$

$$\frac{c}{\Delta p_H(1)} \ge \frac{c}{f\Delta p_H(n-m) + (1-f)\Delta p_L(1)} \qquad \text{iff } f \ge \frac{e^{H-L} - 1}{e^{\alpha(n-m) + H - L} - 1},$$

$$\frac{c}{\Delta p_L(m+1)} \ge \frac{c}{f\Delta p_H(n-m) + (1-f)\Delta p_L(1)} \qquad \text{iff } f \ge \frac{e^{\alpha m} - 1}{e^{\alpha(n-m) + H - L} - 1}.$$

By using the above conditions, we show the optimal bonus in Figure 3.



Figure 3: Optimal Contract with Dis-Figure 4: Optimal Contract with Dispersed Information ($\alpha = 1/2$, n = 5 persed Information ($\alpha = 1/2$, n = 5and m = 2) and m = 3)

Figure 4 shows the comparative statics of the optimal bonus from changing m = 2 (broken curve) to m = 3 (solid curve), without changing n. (so the number of uninformed agents increases, without changing the total number of agents). As m increases, $\frac{c}{\Delta p_L(m+1)}$ decreases, and hence, the candidate equilibrium of the form (1, (1, 0)) becomes easier to eliminate: an informed agent in state L is more incentivized to choose a high effort if the number of the uninformed, who are supposed to choose the high effort in any state, increases. On the other hand, $\frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}$ increases with m, and hence, the candidate equilibrium of the form (0, (1, 0)) becomes harder to eliminate: an uninformed agent is less incentivized to play a high effort if the number of the informed, who are supposed to choose high effort in state H, decreases.

4.3 Comparison

Proposition 7. Full information is never optimal.

Proof. We show that the cost of uniquely implementing e = (1, ..., 1) for any θ under full revelation is always weakly higher than that under no revelation.

Suppose that, given b, e = (1, ..., 1) is a unique Nash equilibrium under full revelation. Then, because e = (0, ..., 0) is not an equilibrium in state L under full information,

$$b > \frac{c}{\Delta p_L(1)}.$$

This implies that $b > \frac{c}{\Delta p_{\phi}(1)}$, and therefore, $e = (1, \ldots, 1)$ is a unique Nash equilibrium under no information.

Proposition 8. Dispersed information is better than full information if and only if

$$\frac{\Delta p_L(m+1) - \Delta p_L(1)}{\Delta p_H(1) - \Delta p_L(1)} \ge f.$$

Proof. Because the (expected) implementation cost is represented by $np_{\phi}(n)b$, we compare b_{FI} and b_{DI} .

If $b_{DI} = \frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}$ or $b_{DI} = \frac{c}{\Delta p_H(1)}$, no information is never optimal because

$$f\Delta p_H(n-m+1) + (1-f)\Delta p_L(1) - \Delta p_\phi(1)$$

= $f[\Delta p_H(n-m+1) - \Delta p_H(1)] \ge 0,$
 $\Delta p_H(1) - \Delta p_\phi(1) = (1-f)[\Delta p_H(1) - \Delta p_L(m+1)] \ge 0,$

which respectively implies $b_{FI} > \frac{c}{f\Delta p_H(m+1)+(1-f)\Delta p_L(1)}$ and $b_{FI} > \frac{c}{\Delta p_H(1)}$. Suppose $b_{DI} = \frac{c}{\Delta p_L(m+1)}$. By rearranging $b_{FI} \ge \frac{c}{\Delta p_L(m+1)}$, we obtain

$$\frac{\Delta p_L(m+1) - \Delta p_L(1)}{\Delta p_H(1) - \Delta p_L(1)} \ge f.$$

As in the two-agent case, the no-information scenario can be optimal only when $b_{DI} = \frac{c}{\Delta p_L(m+1)}$ (or equivalently, $\frac{c}{\Delta p_L(m+1)}$ is higher than $\frac{c}{\Delta p_H(1)}$ and $\frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}$). This means that, as *m* increases, it becomes more likely that $\frac{c}{\Delta p_L(m+1)} < \frac{c}{\Delta p_{\phi}(1)} = b_{NI}$.

In general, b_{DI} is a quasi-convex function of m. If m is sufficiently small, $b_{DI} = \frac{c}{\Delta p_L(m+1)}$, which decreases as *m* increases. As *m* increases, $\frac{c}{f\Delta p_H(n-m+1)+(1-f)\Delta p_L(1)}$ may become higher than $\frac{c}{\Delta p_L(m+1)}$, and after that, b_{DI} is (weakly) increasing in m. Therefore, if the principal can decide how many agents among n agents should be uninformed (i.e., m), the optimal level of m is determined as the point that balances all of the three bonus levels.

Example Suppose $p_{\theta}(x) = \beta e^{\alpha x + \theta}$. Then, $\Delta p_{\theta}(x) = \beta e^{\alpha (x-1) + \theta} (e-1)$. With some manipulation, we obtain the following conditions

$$\frac{\Delta p_L(m+1) - \Delta p_L(1)}{\Delta p_H(1) - \Delta p_L(1)} \ge f \text{ iff } \frac{e^{\alpha m} - 1}{e^{H-L} - 1} \ge f$$

By using the above conditions, we draw Figure 5.

5 **Individual Bonus**

Until the previous section, we has showed that the dispersed information is beneficial for the principal under assumption of the anonymous contract. This result will be reconsiderred in this section under a individual bonus contract, i.e., a contract is a pair (b_i, w_i) such that w_i is paid to agent i if



Figure 5: Dispersed Info. versus No Info. ($\alpha = 1/2, n = 5$ and m = 2)

x = F, and $b_i + w_i$ is paid to agent *i* if x = S. To keep the model simple, we consider n = 2 and m = 1. $B_s(e_1, e_2)$ is defined as a set of a bonus contract in which (e_1, e_2) is an equilibrium under the information structure *s*. Our conclusion in this section is that the benefit of the dispersed information is robust result.

5.1 No information

Lemma 6.

$$B_{NI}(1,1) \setminus \left(\bigcup_{(e_1,e_2)\neq(1,1)} B_{NI}(e_1,e_2) \right) \\ = \left\{ b \mid b_1 > \frac{c}{\Delta p_{\phi}(1)} \text{ and } b_2 > \frac{c}{\Delta p_{\phi}(2)} \right\} \cup \left\{ b \mid b_1 > \frac{c}{\Delta p_{\phi}(2)} \text{ and } b_2 > \frac{c}{\Delta p_{\phi}(1)} \right\}$$

Proof. By subtracting $B_{NI}(0,0)$ from $B_{NI}(1,1)$, we obtain

$$\left\{b \mid b_1 > \frac{c}{\Delta p_{\phi}(1)} \text{ and } b_2 \ge \frac{c}{\Delta p_{\phi}(2)}\right\} \cup \left\{b \mid b_1 \ge \frac{c}{\Delta p_{\phi}(2)} \text{ and } b_2 > \frac{c}{\Delta p_{\phi}(1)}\right\}.$$



Figure 6: BNE in No Information

S: an optimal benchmark contract, A-INI: an optimal anonymous INI contract, INI: an optimal INI contract By subtracting $B_{NI}(1,0)$ and $B_{NI}(0,1)$ from the above set, all the inequalities in the above expression become strict, because the above set overlaps $B_{NI}(1,0)$ (respectively, $B_{NI}(0,1)$) along a line $b_2 = \frac{c}{\Delta p_{\phi}(2)}$ (respectively, $b_1 = \frac{c}{\Delta p_{\phi}(2)}$).

If we allow for individual-specific bonus contracts, $B_s(e_1, e_2)$ is two-dimensional, and the region where each equilibrium exists is given as in Figure 6.

As in the previous sections with anonymous bonus contracts, the lemma states that $B_{NI}(1,1)$ overlaps with $B_{NI}(0,0)$. Hence, under the bonus contract that induces (1,1) as one of the equilibria, (0,0) may be an equilibrium as well. Therefore, the optimal contract uniquely implementing (1,1) must make (0,0) non-equilibrium.

As the figure suggests, the region where (1,1) is the only equilibrium is $B_{NI}(1,1) \setminus B_{NI}(0,0)$, and thus, there are multiple local optima. In the current case, both of them are in fact globally optimal, as in the following proposition.

Proposition 9. An optimal contract is

$$(b_1, b_2) = \left(\frac{c}{\Delta p_{\phi}(2)}, \frac{c}{\Delta p_{\phi}(1)}\right) \text{ or } \left(\frac{c}{\Delta p_{\phi}(1)}, \frac{c}{\Delta p_{\phi}(2)}\right)$$

Implementation cost is $p_{\phi}(2)(\frac{c}{\Delta p_{\phi}(2)} + \frac{c}{\Delta p_{\phi}(1)}).$

The optimality of asymmetric bonus is found by Winter (??) in the context of team production without state uncertainty. In the no information scenario in our model, hence, his result directly applies.

Proof. By lemma 6, the principal faces the following constraints,

$$b \in \left\{ b \mid b_1 > \frac{c}{\Delta p_{\phi}(1)} \text{ and } b_2 > \frac{c}{\Delta p_{\phi}(2)} \right\}$$

or
$$b \in \left\{ b \mid b_1 > \frac{c}{\Delta p_{\phi}(2)} \text{ and } b_2 > \frac{c}{\Delta p_{\phi}(1)} \right\}$$

Because $\left(\frac{c}{\Delta p_{\phi}(1)}, \frac{c}{\Delta p_{\phi}(0)}\right)$ and $\left(\frac{c}{\Delta p_{\phi}(0)}, \frac{c}{\Delta p_{\phi}(1)}\right)$ incur the same implementation cost, we obtain the proposition.

$e_1 \setminus e_2$	(1, 1)	(1,0)	(0,1)	(0, 0)
1	$b_1 \ge \frac{c}{\Delta p_{\phi}(2)}$	$b_1 \ge \frac{c}{f\Delta p_H(2) + (1-f)\Delta_L(1)}$	_	$b_1 \ge \frac{c}{\Delta p_{\phi}(1)}$
	$b_2 \ge \frac{c}{\Delta p_L(2)}$	$\frac{c}{\Delta p_L(2)} \ge b_2 \ge \frac{c}{\Delta p_H(2)}$		$\frac{c}{\Delta p_H(2)} \ge b_2$
0	$b_1 \le \frac{c}{\Delta p_{\phi}(2)}$	$b_1 \le \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}$	_	$b_1 \le \frac{c}{\Delta p_{\phi}(1)}$
	$b_2 \ge \frac{c}{\Delta p_L(1)}$	$\frac{c}{\Delta p_L(1)} \ge b_2 \ge \frac{c}{\Delta p_H(1)}$		$\frac{c}{\Delta p_H(1)} \ge b_2$

Table 1: Conditions of NE: Dispersed Information

• "_" represents no equilibrium.

When asymmetry in bonus is allowed, in order to avoid the low-effort equilibrium, it is enough to give a sufficiently high bonus for just one of the players, $\frac{c}{\Delta_{\phi}(1)}$. In fact, such an agent chooses the high effort as his dominant action (i.e., regardless of the other agent's action).

Given this, it is enough to pay just $\frac{c}{\Delta_{\phi}(2)}$ to the other agent, instead of $\frac{c}{\Delta_{\phi}(1)}$, which is a difference from the anonymous bonus case.

5.2 Dispersed information

Lemma 7. 1. $B_{DI}(1, (0, 1)) = B_{DI}(0, (0, 1)) = \emptyset$.

- 2. $B_{DI}(1,(1,1)) \cap B_{DI}(1,(0,0)) = \emptyset$.
- 3. $B_{DI}(1, (1, 1)) \cap B_{DI}(0, (0, 0)) = \emptyset$ if and only if $\Delta p_H(1) > \Delta p_L(2)$.

As in the previous sections, the first statement says that there is no equilibrium where the informed agent works only in the low state. The second statement says that we can ignore (1, (0, 0)) as a candidate equilibrium in investigating the optimal contract. The last statement identifies the condition under which we can ignore (0, (0, 0)) as a candidate equilibrium in investigating the optimal contract.

Proof. 1. $B_{DI}(1, (0, 1)) = B_{DI}(0, (0, 1)) = \emptyset$. Suppose that $(e_2(H), e_2(L)) = \emptyset$



Figure 7: BNE if $\Delta p_L(2) < \Delta p_H(1)$ Figure 8: BNE if $\Delta p_L(2) \ge \Delta p_H(1)$ S: an optimal benchmark contract, A-INI: an optimal anonymous INI contract, C0,C1,C2,C3: an optimal INI contract

(0, 1). Then

$$\frac{c}{\Delta p_H(e_1+1)} \ge b_2 \ge \frac{c}{\Delta p_L(e_1)}.$$

However, there is no b_2 satisfying the above inequality because $\Delta p_H(e_1 + 1) \geq \Delta p_L(e_1)$.

2. $B_{DI}(1,(1,1)) \cap B_{DI}(1,(0,0)) = \emptyset$

$$B_{DI}(1, (1, 1)) \cap B_{DI}(1, (0, 0)) = \left\{ b \mid b_1 \ge \frac{c}{\Delta p_{\phi}(1)} \text{ and } \frac{c}{\Delta p_H(2)} \ge b_2 \ge \frac{c}{\Delta p_L(2)} \right\}$$

The above is empty because $\Delta p_H(2) \geq \Delta p_L(2)$.

3. $B_{DI}(1,(1,1)) \cap B_{DI}(0,(0,0)) = \emptyset$ if and only if $\Delta p_H(1) > \Delta p_L(2)$.

$$B_{DI}(1, (1, 1)) \cap B_{DI}(0, (0, 0)) = \left\{ b \mid \frac{c}{\Delta p_{\phi}(1)} \ge b_1 \ge \frac{c}{\Delta p_{\phi}(2)} \text{ and } \frac{c}{\Delta p_H(1)} \ge b_2 \ge \frac{c}{\Delta p_L(2)} \right\}$$

If $\Delta p_H(1) > \Delta p_L(2)$, the above set is empty.

The first statement simply says that it is impossible for the informed agent to work only in the low state but in the high state. The second statement says that, for unique implementation of (1, (1, 1)), it is not a binding constraint to make (1, (0, 0)) non-equilibrium.

The third statement says that, when the state effect is large $(\Delta p_H(1) > \Delta p_L(2))$, then for unique implementation of of (1, (1, 1)), making (0, (0, 0))non-equilibrium is not a binding constraint. To see this, note that, to make (1, (1, 1)) an equilibrium, it is necessary to satisfy $b_2 \geq \frac{c}{\Delta p_L(2)}$ so that the informed works in the low state given the uninformed works, while to make (0, (0, 0)) non-equilibrium, $b_2 \geq \frac{c}{\Delta p_H(1)}$ is sufficient. Because $\Delta p_H(1) > \Delta p_L(2)$, if (1, (1, 1)) is made an equilibrium, then (0, (0, 0)) cannot be an equilibrium.

Figure 7 and 8 illustrate the regions of bonus contracts where each effort profile is an equilibrium. Our optimal bonus contract would be the lower-left point of the region where (1, (1, 1)) is an equilibrium while any other effort profile is not.

Lemma 8. • If $\Delta p_H(1) > \Delta p_L(2)$,

$$B_{DI}(1,1) \setminus \left(\bigcup_{(e_1,e_2) \neq (1,1)} B_{DI}(e_1,e_2) \right) \\ = \left\{ b \mid b_1 > \frac{c}{\Delta p_{\phi}(2)} \text{ and } b_2 > \frac{c}{\Delta p_L(1)} \right\} \\ \cup \left\{ b \mid b_1 > \frac{c}{f \Delta p_H(2) + (1-f) \Delta p_L(1)} \text{ and } b_2 > \frac{c}{\Delta p_L(2)} \right\}.$$

• If $\Delta p_H(1) \leq \Delta p_L(2)$,

$$B_{DI}(1,1) \setminus \left(\bigcup_{(e_1,e_2) \neq (1,1)} B_{DI}(e_1,e_2) \right)$$

= $\left\{ b \mid b_1 > \frac{c}{\Delta p_{\phi}(2)} \text{ and } b_2 > \frac{c}{\Delta p_L(1)} \right\}$
 $\cup \left\{ b \mid b_1 > \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)} \text{ and } b_2 > \frac{c}{\Delta p_H(1)} \right\}$
 $\cup \left\{ b \mid b_1 > \frac{c}{\Delta p_{\phi}(1)} \text{ and } b_2 > \frac{c}{\Delta p_L(2)} \right\}.$

Proof. Suppose that $\Delta p_H(1) > \Delta p_L(2)$. By Lemma 7, we have to consider $B_{DI}(0, (1, 0)), B_{DI}(1, (1, 0))$ and $B_{NI}(0, (1, 1))$. By subtracting $B_{DI}(0, (1, 0))$ from $B_{DI}(1, (1, 1))$, we obtain

$$\left\{ b \mid b_1 \ge \frac{c}{\Delta p_{\phi}(2)} \text{ and } b_2 > \frac{c}{\Delta p_L(1)} \right\}$$
$$\cup \left\{ b \mid b_1 > \frac{c}{f \Delta p_H(2) + (1 - f) \Delta p_L(1)} \text{ and } b_2 \ge \frac{c}{\Delta p_L(2)} \right\}.$$

By subtracting $B_{DI}(1, (1, 0))$ and $B_{NI}(0, (1, 1))$ from the above set, all the inequalities in the above expression become strict, because the above set overlaps $B_{DI}(1, (1, 0))$ (respectively, $B_{NI}(0, (1, 1))$) along a line $b_2 = \frac{c}{\Delta p_L(2)}$ (respectively, $b_1 = \frac{c}{\Delta p_{\phi}(2)}$).

Suppose that $\Delta p_H(1) \leq \Delta p_L(2)$. By Lemma 7, we have to consider $B_{DI}(0, (0, 0)), B_{DI}(0, (1, 0)), B_{DI}(1, (1, 0))$ and $B_{NI}(0, (1, 1))$. By subtracting $B_{DI}(0, (0, 0))$ from $B_{DI}(1, (1, 1))$, we obtain

$$\left\{b \mid b_1 \ge \frac{c}{\Delta p_{\phi}(2)} \text{ and } b_2 > \frac{c}{\Delta p_H(1)}\right\} \cup \left\{b \mid b_1 > \frac{c}{\Delta p_{\phi}(1)} \text{ and } b_2 \ge \frac{c}{\Delta p_L(2)}\right\}$$

By subtracting $B_{DI}(0, (1, 0))$ from the above set, we obtain

$$\left\{ b \mid b_1 \ge \frac{c}{\Delta p_{\phi}(2)} \text{ and } b_2 > \frac{c}{\Delta p_L(1)} \right\}$$
$$\cup \left\{ b \mid b_1 > \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)} \text{ and } b_2 > \frac{c}{\Delta p_H(1)} \right\}$$
$$\cup \left\{ b \mid b_1 > \frac{c}{\Delta p_{\phi}(1)} \text{ and } b_2 \ge \frac{c}{\Delta p_L(2)} \right\}.$$

By subtracting $B_{DI}(1, (1, 0))$ and $B_{NI}(0, (1, 1))$ from the above set, all the inequalities in the above expression become strict.

The lemma shows that, regardless of the size of the state effect, the optimal bonus contract is characterized by the three conditions: making (1, (1, 1))an equilibrium, preventing (0, (1, 0)) from being an equilibrium, and preventing (0, (0, 0)) from being an equilibrium. Furthermore, the first statement of the lemma shows that, if the state effect is large (i.e., $\Delta p_H(1) > \Delta p_L(2)$), then the constraint of preventing (0, (0, 0)) from being an equilibrium is not binding. This is because, when the state effect is larger, then it is easier to make the informed agent work in the high state (hence making (0, (0, 0))non-equilibrium).

Because the region of bonus contracts that satisfy those conditions is not convex, there are multiple local optima. The next proposition characterizes the optimal contract.

Proposition 10. 1. If $\Delta p_H(1) > \Delta p_L(2)$, an optimal contract is

$$b^{C0} = \left(\frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}, \frac{c}{\Delta p_L(2)}\right)$$

Implementation cost is $p_{\phi}(2)\left(\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}+\frac{c}{\Delta p_L(2)}\right)$.

2. If $\Delta p_H(1) \leq \Delta p_L(2)$, an optimal contract is $b_{DI} = \underset{b \in \{b^{C1}, b^{C2}, b^{C3}\}}{\operatorname{arg min}} b_1 + b_2$ where

$$\begin{split} b^{C1} &= (\frac{c}{\Delta p_{\phi}(1)}, \frac{c}{\Delta p_{L}(2)}), \\ b^{C2} &= (\frac{c}{f\Delta p_{H}(2) + (1 - f)\Delta p_{L}(1)}, \frac{c}{\Delta p_{H}(1)}), \\ b^{C3} &= (\frac{c}{\Delta p_{\phi}(2)}, \frac{c}{\Delta p_{L}(1)}). \end{split}$$

Implementation cost is $p_{\phi}(2) \min_{b \in \{b^{C1}, b^{C2}, b^{C3}\}} b_1 + b_2$.

Proof. Suppose $\Delta p_H(1) > \Delta p_L(2)$. By Lemma 8, the principal faces the following constraints, i.e.,

$$b \in \left\{ b \mid b_1 > \frac{c}{\Delta p_{\phi}(2)} \text{ and } b_2 > \frac{c}{\Delta p_L(1)} \right\}$$

or
$$b \in \left\{ b \mid b_1 > \frac{c}{f \Delta p_H(2) + (1 - f) \Delta p_L(1)} \text{ and } b_2 > \frac{c}{\Delta p_L(2)} \right\}.$$

Because the implementation cost is increasing with respect to b_i , the optimal bonus is either $\left(\frac{c}{\Delta p_{\phi}(2)}, \frac{c}{\Delta p_L(1)}\right)$ or $\left(\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}, \frac{c}{\Delta p_L(2)}\right)$. By comparing the implementation costs, we obtain

$$\begin{aligned} &\frac{c}{\Delta p_{\phi}(2)} + \frac{c}{\Delta p_{L}(1)} - \left[\frac{c}{f\Delta p_{H}(2) + (1 - f)\Delta p_{L}(1)} + \frac{c}{\Delta p_{L}(2)}\right] \\ &= \frac{f(1 - f)\left[\Delta p_{L}(2) - \Delta p_{L}(1)\right]\left[\Delta p_{H}(2) + \Delta p_{L}(2)(\Delta p_{H}(2) - \Delta p_{L}(1))\right]}{\Delta p_{L}(1)\Delta p_{L}(2)[f\Delta p_{H}(2) + (1 - f)\Delta p_{L}(1)][f\Delta p_{H}(2) + (1 - f)\Delta p_{L}(2)]} \\ &\geq 0. \end{aligned}$$

Therefore,
$$\left(\frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}, \frac{c}{\Delta p_L(2)}\right)$$
 is optimal.

First, consider the case where the state effect is large (i.e., $\Delta p_H(1) > \Delta p_L(2)$). The point C0 in Figure 7 corresponds to the optimal bonus contract in this case. As discussed above, in this case, the optimal bonus contract is characterized by the two conditions: making (1, (1, 1)) an equilibrium, and preventing (0, (1, 0)) from being an equilibrium. In order to make (0, (1, 0))non-equilibrium, the contract C0 makes the uninformed agent work given that the informed agent works only in the high state. The contract then makes the informed agent work given that the uninformed agent works (in any state).

To see this is optimal, consider the alternative way to make (1, (1, 1))the unique equilibrium: pay $\frac{c}{\Delta p_L(1)}$ to the informed agent so that he is incentivized to work in any state even if the uninformed does not work, and then pay $\frac{c}{\Delta p_{\Delta}(2)}$ to the uninformed so that he is incentivized to work given the informed works in any state. The difference of these two implementation costs is

$$-[\frac{c}{\Delta p_L(1)} - \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}] + [\frac{c}{\Delta p_L(2)} - \frac{c}{\Delta p_{\phi}(2)}],$$

where C0 is optimal if this is negative. The first two terms correspond to the difference in the cost of making (0, (1, 0)) non-equilibrium. Such a cost is smaller in C0, because it is easier to make the uninformed work given that the informed agent works in the high state than to make the informed work in any state given the uninformed does not work. On the other hand, as in the last two terms, to make (1, (1, 1)) the unique equilibrium, C0 must pay $\frac{c}{\Delta p_L(2)}$ to the informed agent, while the alternative contract only needs to pay $\frac{c}{\Delta p_{\phi}(2)}$ to the uninformed. When the state effect is large, the effect of the first two terms dominates, making C0 optimal.

Next, consider the case where the state effect is small (i.e., $\Delta p_H(1) < \Delta p_L(2)$). As shown in the Figure 8, there are three candidate bonus pairs as the optimal contract, C1, C2, and C3.

C1 pays $\frac{c}{\Delta p_{\phi}(1)}$ to the uninformed agent so that he works even if the informed does not, and then pays $\frac{c}{\Delta p_L(2)}$ to the informed agent so that he works in the low state if the uninformed works. C3 is similar to C1 but the opposite. It pays $\frac{c}{\Delta p_L(1)}$ to the uninformed so that he works even if the uninformed does not, and then pays $\frac{c}{\Delta p_{\phi}(2)}$ to the uninformed. As opposed to the no information scenario, C1 and C3 may induce different implementation costs. Roughly, if $\Delta p_{\phi}(2) - \Delta p_{\phi}(1)$ is sufficiently lower than $\Delta p_L(2) - \Delta p_L(1)$ (or equivalently, if $\Delta p_H(2) - \Delta p_H(1)$ is sufficiently lower than $\Delta p_L(2) - \Delta p_L(2) - \Delta p_L(1)$), then C1 dominates C3, and vice versa. The quantity $\Delta p_{\theta}(2) - \Delta p_{\theta}(1)$ can be interpreted as the size of complementarity of efforts in each state θ .

Comparison with C2 is more involved. C2 pays $\frac{c}{\Delta p_H(1)}$ to the informed agent so that he works in the high state even if the uninformed does not. Then it pays $\frac{c}{f\Delta p_H(2)+(1-f)\Delta p_L(1)}$ to the uninformed agent so that he works if the informed works at least in the high state. Now given this, the informed

agent is automatically incentivized to work in the low state as well, because his bonus $\frac{c}{\Delta p_H(1)}$ is higher than $\frac{c}{\Delta p_L(2)}$, a bonus level above which the informed works in the low state if the uninformed works.

Here, we only compare C2 and C3. Comparison between C1 and C2 are intuitively similar (though subtly different), and so omitted. The difference in the implementation costs is

$$-\left[\frac{c}{\Delta p_L(1)} - \frac{c}{f\Delta p_H(2) + (1-f)\Delta p_L(1)}\right] + \left[\frac{c}{\Delta p_H(1)} - 0\right] - \left[\frac{c}{\Delta p_\phi(2)} - 0\right],$$

where C2 is better than C3 if this is negative, and vice versa. The first two terms correspond to the difference in the cost of making (0, (1, 0)) nonequilibrium. Such a cost is smaller in C2, because it is easier to make the uninformed work given that the informed agent works in the high state (as in C2) than to make the informed work in any state given the uninformed does not work (as in C3).

The middle two terms correspond to the difference in the cost of making (0, (0, 0)) non-equilibrium. C2 pays $\frac{c}{\Delta p_H(1)}$ to the informed agent so that he works in the high state. Such an additional payment is unnecessary in C3, because in C3, by paying $\frac{c}{\Delta p_L(1)}$ to the informed, both (0, (1, 0)) and (0, (0, 0)) are made non-equilibrium at the same time.

The last two terms correspond to the difference in the cost of making (0, (0, 0)) a (unique) equilibrium. C3 pays $\frac{c}{\Delta p_{\phi}(2)}$ to the uninformed agent so that he works if the uninformed works. Such an additional payment is unnecessary in C2, because in C2, by paying $\frac{c}{\Delta p_H(1)} (> \frac{c}{\Delta p_L(2)})$ to the informed, (1, (1, 1)) is already an equilibrium.

Example Suppose $p_{\theta}(x) = \beta e^{\alpha x + \theta}$. Then, $\Delta p_{\theta}(x) = \beta e^{\alpha (x-1) + \theta} (e-1)$. C0 is optimal if

$$\frac{c}{\Delta p_L(2)} \ge \frac{c}{\Delta p_H(1)} \text{ iff } H - L \ge \alpha.$$

Suppose that $H - L < \alpha$. In this situation, C3 is dominated by C1 because $b_1^{C3} + b_2^{C3} \ge b_1^{C1} + b_2^{C1}$ iff $f \ge -\frac{1}{e^{H-L}}$. C1 is optimal if

$$[\frac{c}{fe^{\alpha+H}+(1-f)e^L}-\frac{c}{e^L}] \ge [\frac{c}{fe^{\alpha+H}+(1-f)e^{\alpha+L}}-\frac{c}{\Delta e^H}]$$

where the left-hand side represents "indirect effect" and the right-hand side "multi-object effect." We obtain the following Figure 9



Figure 9: Optimal Contract in Dispersed Information ($\alpha = 1/2$)

As in the previous proposition, the optimal contract sharply differs depending on the relative magnitude of the state effect and the effort effect. If the state effect is larger (i.e., $H - L > \alpha = 0.5$), then C0 is the optimal bonus contract.

On the other hand, if the effort effect is larger (i.e., $H - L < \alpha = 0.5$), the optimal bonus contract is either C1, C2, or C3. In this exponential example, as in Figure 9, C3 is never optimal. C1 is optimal if f and H - L are large.

5.3 Comparison

Proposition 11. Full information is never optimal.

Proof. We show that the cost of uniquely implementing e = (1, 1) for any θ under full revelation is always weakly higher than that under no revelation.

Suppose that, given (b_1, b_2) , e = (1, 1) is a unique Nash equilibrium under full revelation. First, for $\theta = L$, for each i = 1, 2,

$$b_i \ge \frac{c}{\Delta p_L(2)},$$

so that e = (1, 1) is one of the equilibria. This implies that e = (1, 1) is an equilibrium with no information, because

$$\frac{c}{\Delta p_L(2)} \ge \frac{c}{\Delta p_\phi(2)},$$

and hence $b_i \geq \frac{c}{\Delta p_{\phi}(2)}$ for each *i*.

Second, because e = (0,0) is not an equilibrium, there is at least one agent, say 1, where

$$b_1 > \frac{c}{\Delta p_L(1)},$$

and because e = (1, 0) is not an equilibrium either, for agent 2,

$$b_2 > \frac{c}{\Delta p_L(2)}.$$

These imply, because $\frac{c}{\Delta p_L(x)} \ge \frac{c}{\Delta p_{\phi}(x)}$ for x = 1, 2,

$$b_1 > \frac{c}{\Delta p_{\phi}(1)},$$

$$b_2 > \frac{c}{\Delta p_{\phi}(2)}.$$

The first inequality implies that e = (0, 0) is not an equilibrium. This also implies that e = (0, 1) is not an equilibrium, because $b_1 > \frac{c}{\Delta p_{\phi}(1)} \ge \frac{c}{\Delta p_{\phi}(2)}$. The second inequality implies that e = (1, 0) is not an equilibrium.

Therefore, (b_1, b_2) uniquely implements e = (1, 1) under no information.

Proposition 12. Dispersed information is least costly if and only if

$$\frac{1}{\Delta p_{\phi}(2)} + \frac{1}{\Delta p_{\phi}(1)} \ge \frac{1}{f \Delta p_{H}(2) + (1 - f) \Delta p_{L}(1)} + \frac{1}{\min\{\Delta p_{L}(2), \Delta p_{H}(1)\}}.$$

Proof. We compare no information case with dispersed information case. Suppose $\Delta p_H(1) > \Delta p_L(2)$. In this case, C1 and C3 are dominated by the optimal contract under no information because

$$(C1) - (\text{no info.}) = \frac{f(\Delta p_H(2) - \Delta p_L(2))}{\Delta p_\phi(2)\Delta p_L(2)} \ge 0,$$

(C3) - (no info.) =
$$\frac{f(\Delta p_H(1) - \Delta p_L(1))}{\Delta p_\phi(1)\Delta p_L(1)} \ge 0.$$

As should be clear from the definitions, C1 and C3 are dominated by the no-information optimal contract. For any given choice of the uninformed agent, the cost of making the informed work in the low state is higher than that in the average state, where the latter corresponds to the no-information case.

Therefore, the dispersed-information scenario is optimal only when C0 or C2 is used. To understand the condition in this proposition, we define the difference in the implementation costs in the no-information and dispersed-information scenarios. Because C1 and C3 do not play any role in this comparison, the implementation cost in the dispersed-information scenario is simply defined by the minimum of C0 and C2. This is without loss of generality as far as we are concerned with the sign of the difference D.

$$D \equiv \frac{1}{f\Delta p_H(2) + (1 - f)\Delta p_L(1)} + \frac{1}{\min\{\Delta p_L(2), \Delta p_H(1)\}} - \frac{1}{\Delta p_\phi(2)} + \frac{1}{\Delta p_\phi(1)}$$

D < 0 (or D > 0) means that dispersed information is better (or worse).

This equation is decomposed to

$$D = \begin{bmatrix} \frac{c}{\Delta p_L(2)} - \frac{c}{\Delta p_{\phi}(2)} \end{bmatrix}$$
$$- \begin{bmatrix} \frac{c}{\Delta p_{\phi}(1)} - \frac{c}{\Delta p_{\phi}(2)} & - \underbrace{\max\left\{0, \frac{c}{\Delta p_H(1)} - \frac{c}{\Delta p_L(2)}\right\}}_{\text{a cost to avoid no effort eq. under no info.} & \text{a cost to avoid no effort eq. under dispersed info.} \\+ \begin{bmatrix} \frac{1}{f\Delta p_H(2) + (1 - f)\Delta p_L(1)} - \frac{1}{\Delta p_{\phi}(2)} \end{bmatrix}.$$

The first two terms correspond to the difference in the cost of making (1, (1, 1)) an equilibrium. The sign is positive, meaning that it is always easier in the no-information scenario to make (1, (1, 1)) an equilibrium. We already saw this effect in the benchmark case in Section 2 (although we only considered anonymous contracts there, the intuition is the same).

The second two terms correspond to the difference in the cost of preventing "no effort in any state" from being an equilibrium. In C0 and C2, this is made possible by paying $\frac{c}{\Delta p_H(1)}$ to the informed agent, so that he works in the high state even if the uninformed does not work. This is lower than the corresponding cost in the no-information scenario (hence the second two terms is negative), because no agent observes the true state.

The last two terms correspond to the cost of making (0, (1, 0)) nonequilibrium, which is only relevant in the dispersed-information case (and hence is positive).

Therefore, the dispersed-information scenario would be better if the second effect dominates the other two. We illustrate it in the example below. **Example** Suppose $p_{\theta}(x) = \beta e^{\alpha x + \theta}$. Then, $\Delta p_{\theta}(x) = \beta e^{\alpha(x-1)+\theta}(e-1)$. By some manipulation, we obtain the following conditions

 $\frac{1}{fe^{\alpha+H} + (1-f)e^{\alpha+L}} + \frac{1}{fe^H + (1-f)e^L} \ge \frac{1}{fe^{\alpha+H} + (1-f)e^L} + \frac{1}{e^{\min\{\alpha+L,H\}}}.$

This is illustrated in Figure 9.



Figure 10: Optimal Information allocation ($\alpha = 1/2$)

As in the anonymous-contract case, if the state effect H - L is large as well as f, the no-information scenario becomes dominant. On the other hand, with individual contracts, the no-information scenario is also dominant when the state effect is small as well as f. This is because of a new effect in the individual-contract case: preventing (0, (1, 0)) from being an equilibrium is now too costly.

The following proposition provides comparative statics.

Proposition 13. Assume $\Delta p_L(2) < \Delta p_H(1)$. Then, there exist $(q^*, r^*, s^*, t^*, f^*)$ such that

- (i): D < 0 if $\Delta p_L(1) < q^*$. D > 0 if $\Delta p_L(1) > q^*$. $q^* \in [0, \Delta p_L(2))$. $\frac{dD}{d\Delta p_L(1)} > 0.^8$
- (ii): D > 0 if $\Delta p_L(2) < r^*$. D < 0 if $\Delta p_L(2) > r^*$. $r^* \in (\Delta p_L(1), \Delta p_H(1))$.
- (iii): D < 0 if $\Delta p_H(1) < s^*$. D > 0 if $\Delta p_H(1) > s^*$. $s^* \in (\Delta p_L(2), \Delta p_H(2))$.
- (iv): D > 0 if $\Delta p_H(2) < t^*$. D < 0 if $\Delta p_H(2) > t^*$. $t^* \in (\Delta p_H(1), 1]$. $\frac{dD}{dz} < 0$.

(v):
$$D < 0$$
 if $f < f^*$. $D > 0$ if $f > f^*$. $f^* \in [0, 1)$.

Proof. To avoid a redundancy of notation, we set

$$q = \Delta p_L(1), \qquad r = \Delta p_L(2),$$

$$s = \Delta p_H(1), \qquad t = \Delta p_H(2).$$

Note that the sign of D is the same as the sign of E, where

$$E = (r + ft + (1 - f)q)(ft + (1 - f)r)(fs + (1 - f)q)$$

-r(fs + ft + (1 - f)r + (1 - f)q)(ft + (1 - f)q).

(i) For q:

It is straightforward that E > 0 if q = r, and that $\frac{dD}{dq} > 0$. Therefore, either E is positive for all $q \in (0, r)$, or there exists a unique $q^* \in (0, r)$ such that E < 0 for $q \in (0, q^*)$ and E > 0 for $q \in (q^*, r)$.

(ii) For r:

It is straightforward that E > 0 if r = q and E < 0 if r = s. Because E is a quadratic function of r, there exists a unique $r^* \in (q, s)$ such that E < 0 for $r \in (q, r^*)$ and E > 0 for $y \in (r^*, s)$.

⁸Note that there may not be any value of q^* such that D > 0. This is represented by the case where $q^* = 1$. Similar for the other cases.

(iii) For s:

It is straightforward that E < 0 if s = r and E > 0 if s = t. Because E is a linear function of s, there exists a unique $s^* \in (r, t)$ such that E < 0 for $s \in (r, s^*)$ and E > 0 for $s \in (s^*, t)$.

(iv) For t:

It is straightforward that E > 0 if t = s, and that $\frac{dD}{dt} < 0$. Therefore, either E is positive for all $t \in (s, 1)$, or there exists a unique $t^* \in (s, 1)$ such that E > 0 for $t \in (s, t^*)$ and E < 0 for $t \in (t^*, 1)$.

(v) For f:

It is straightforward that E = 0 if f = 0 and E > 0 if f = 1. Also, we have E = fF where

$$F = f^{2}(t-q)(t-r)(s-q) + f(t-r)[(s-q)(q+r) - (t-q)(r-q)] + q^{2}(t-r) - r^{2}(t-s).$$

Thus, in the following, we investigate the sign of F. Because F > 0 at f = 1, there are three possibilities: (I) F > 0 for all $f \in (0, 1)$, (II) F < 0 at f = 0, or (III) F > 0 at f = 0 but there is some $f \in (0, 1)$ at which F < 0. We show that case (III) does not happen.

Because the coefficient of f^2 , (t-q)(t-r)(s-q), is positive, if the coefficient of f, or more precisely, if (s-q)(q+r) - (t-q)(r-q) is positive, then we are in case (I). So suppose that $(s-q)(q+r) \leq (t-q)(r-q)$. If the constant term $q^2(t-r) - r^2(t-s)$ is positive at the same time, then we are in case (III) (otherwise we are in case (II)). However,

$$q^{2}(t-r) - r^{2}(t-s) = t(q^{2} - r^{2}) - q^{2}r + r^{2}s$$

$$\leq r^{2}s - q^{2}r + (q+r)[(s-q)(q+r) - q(r-q)]$$

$$= q(q(r-s) + 2(q^{2} - rs)) < 0.$$

Proposition 14. Assume $\Delta p_L(2) > \Delta p_H(1)$. Then, there exist $(q^*, r^*, s^*, t^*, f^*)$ such that

(i) D < 0 if $\Delta p_L(1) < q^*$. D > 0 if $\Delta p_L(1) > q^*$. $q^* \in (0, \Delta p_H(1))$. $\frac{dD}{d\Delta p_L(1)} > 0.$

(ii)
$$D > 0$$
 if $\Delta p_H(1) < s^*$. $D < 0$ if $\Delta p_H(1) > s^*$. $s^* \in (\Delta p_L(1), \Delta p_L(2))$.

- (iii) D < 0 if $\Delta p_L(2) < r^*$. D > 0 if $\Delta p_L(2) > r^*$. $r^* \in (\Delta p_H(1), \Delta p_H(2)]$.
- (iv) D > 0 if $\Delta p_H(2) < t^*$. D < 0 if $\Delta p_H(2) > t^*$. $t^* \in [\Delta p_L(2), 1]$. $\frac{dD}{d\Delta p_H(2)} < 0.$
- (v) D > 0 if $f < f^*$. D < 0 if $f > f^*$. $f^* \in (0, 1]$.

Proof. To avoid a redundancy of notation, we set

$$q = \Delta p_L(1), \qquad r = \Delta p_L(2),$$

$$s = \Delta p_H(1), \qquad t = \Delta p_H(2).$$

The sign of D is the same as the sign of E', where

$$E' = (s + ft + (1 - f)q)(ft + (1 - f)r)(fs + (1 - f)q)$$

-s(fs + ft + (1 - f)r + (1 - f)q)(ft + (1 - f)q).

(i) For q:

It is straightforward that E < 0 if s = 0 and E > 0 if q = s, and that $\frac{dD}{dq} > 0$. Therefore, there exists a unique $q^* \in (0, s)$ such that E' < 0 for $q \in (0, q^*)$ and E' > 0 for $q \in (q^*, s)$.

(ii) For s:

It is straightforward that E' > 0 if s = q and E' < 0 if s = r. Because E' is a quadratic function of s, there exists a unique $s^* \in (q, r)$ such that E' < 0 for $s \in (q, s^*)$ and E' > 0 for $s \in (s^*, r)$.

(iii) For r:

It is straightforward that E' < 0 if r = s. Because E' is a linear function of r, either E' is positive for all $r \in (s, t)$, or there exists a unique $r^* \in (s, t)$ such that E' < 0 for $r \in (s, r^*)$ and E' > 0 for $r \in (r^*, t)$. (iv) For t:

It is straightforward that $\frac{dD}{dt} < 0$. Therefore, either E' is positive for all $t \in (s, 1)$, E' is negative for all $t \in (s, 1)$, or there exists a unique $t^* \in (s, 1)$ such that E' > 0 for $t \in (r, t^*)$ and E' < 0 for $t \in (t^*, 1)$.

(v) For f:

It is straightforward that E' > 0 if f = 0 and E' = 0 if f = 1. Also, we have E' = (1 - f)F' where, letting g = 1 - f,

$$F' = -g^{2}(t-r)(s-q)(t-q) + g(s-q)[(t-r)(t+s) + (t-q)(t-s)] + s^{2}(r-q) - t^{2}(s-q).$$

Because F' > 0 at g = 1, and the coefficient of g^2 is negative, either F' is positive for all $f \in (0, 1)$, or there is a unique $f^* \in (0, 1)$ such that F' > 0 for $f \in (0, f^*)$ and F' < 0 for $f \in (f^*, 1)$.

6 Conclusion

In this paper, we studied an optimal organization structure in terms of information allocation. In a unique implementation problem of desirable effort levels in the context of team production, we found an important channel through which information structure affects implementation cost. Under certain conditions, this channel makes it optimal to asymmetrically inform the agents, even if they are ex ante symmetric.

Although we made a number of simplifying assumptions, we believe that the simple intuition found in this paper could be generalized to more involved situations. Also, although we focused on ex ante symmetric agents to highlight the intrinsic motivation of endogenous asymmetric information among agents, we believe that it would be interesting how this effect would interact with the other sources of asymmetry of agents in terms of their characteristics, tasks, (exogenous) information, and so on. These are left open for future research.

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