# Strategic Voting and Non-asymptotic Condorcet Jury Theorem: A Sufficient Condition for Superiority of Single Person Decision-making

Naruto Nagaoka*

August 5, 2015

**Abstract** The Condorcet jury theorem claims under certain conditions that committees with more members can decide more efficiently. We analyze the efficiency of decision under strategic voting and we compare the efficiency between the single person decision-making and the group decision-making by voting. We construct a Bayesian model in which the degree of information precision of a member is determined randomly. Our main result is that the single person decision-making is more efficient than group decision-making by voting under certain conditions. Moreover, we establish a sufficient condition under which the single person decision-making is superior to decision-making by voting in any sub-committee selected from the group including the group itself.

**JEL Classification Numbers:** D71, D72, D82

**Keywords:** Condorcet jury theorem, decision-making in committees, strategic voting

*Graduate School of Business Administration, Kobe University; nagaoka@gold.kobe-u.ac.jp

# 1 Introduction

The design of decision procedure to aggregate individuals' information and make better decisions is the most basic problem of collective decision-making. Voting is one of the most basic procedure of collective decision-making, and is employed in diverse processes in societies. There is a well-known classical result on voting and information aggregation, called the Condorcet jury theorem. Consider decision-making in committees under uncertainty as follows. Suppose that there are two alternatives and that the committee with $2n+1$ members must decide to choose one of the alternatives. One is better than the other for all members, but no member knows which one is better. It is assumed that each member has partial information about which one is better. The Condorcet jury theorem states the relationship between the probability that the committee chooses the better alternative (hereafter, decision probability) and the committee size. Suppose that the decision is made by simultaneous voting with the simple majority rule, that is, all the members participate the decision-making and each member has an equal vote. If the probability that each member votes for the better alternative (hereafter, voting probability) is larger than $1/2$, then, [1] the decision probability is increasing in the committee size (monotonicity), [2] the decision probability goes to one as the committee size goes to infinity (asymptotic property). The first part of the Condorcet jury theorem implies that [3] it is better to have all the members receive the right to vote and participate the decision than to let a particular person make a decision. The third part of the Condorcet jury theorem provides the rationale for group decision-making by voting, because it establishes that the group can make better decision than single person (superiority of group decision-making by voting over single person decision-making).

In the classical Condorcet jury theorem, it is assumed that the voting probability of each member is exogenously given and fixed. In the simplest version of the Condorcet jury theorem, each member's voting probability is the same and their voting probabilities for the better alternative are independent. Several authors have examined the robustness of this assumption with respect to the superiority of group decision-making by voting over single person decision-making. Ben-Yashar and Paroush (2000) analyzed a model in which voting probabilities are different across members. They showed that if all the member's voting probabilities for the better alternative are larger than $1/2$, the probability that the committee chooses the better alternative is larger than the probability that a person who is randomly selected as the decision-maker chooses the better alternative. Their result indicates that the superiority of group decision-making by voting over single person decision-making also holds under the heterogeneity of member's competence (which reflects their informational abilities). Wit (1998) analyzed a model in which each member follows strategic voting behavior; each member predicts the other members' voting behaviors and chooses his vote so as to maximize his expected utility from the resulting committee's decision. He showed that the superiority of group decision-making by voting over single person decision-making also holds under the strategic voting behavior. All these studies seem to point to a conclusion that the superiority of group decision-making by voting over single person decision-making is robust.

In contrast to them, we show that the superiority of group decision-making by voting over single person decision-making does not hold once the members are endowed with heterogenous informational abilities *and* they vote strategically without common knowledge of the informational abilities. In this paper, we provide a sufficient condition for the superiority of single person decision-making over group decision-making.

The model of decision-making in committees we analyze is as follows. The committee must choose one of two alternatives. There are two states and each of the states represents which is the better alternative. Each committee member receives a binary signal about the state. We assume that a member receives a signal with some degrees of information precision. There are two types of the degree of information precision; higher type and lower type. The degree of information precision of a member is determined randomly. Then, the member receives a binary signal with his degree of information precision. We assume that the degree of information precision and the signal become his private information. We consider two procedures for the committee decision; the single person decision-making and group decision-making by voting. When the decision procedure is group decision-making by voting, they vote simultaneously after receiving the signals and the committee decision is made by the simple majority rule. When the decision procedure is single person decision-making, a member who had been randomly selected in advance as the decision-maker chooses an alternative.

Our result is that the single person decision-making is superior to the group decision-making by voting when the parameters in our model satisfy the following conditions; [1] the degree of information precision of the higher type is sufficiently high, [2] the probability distribution of the degree of information precision is biased for the lower type, [3] the prior probability distribution of the state is biased toward one of the states. More precisely, there exists an interval of prior probabilities for the more likely state under which the superiority of single person decision-making holds under the first and second conditions. This interval is located closer to probability one when the degree of information precision of the higher type is higher. In the extreme case in which the signal received by the member of the higher type is perfect information about the state, the upper bound of the interval is exactly equal to one. In words, the single person decision-making is superior to the group decision-making by voting when the committee with the members who have higher information precision with small probabilities faces decision problems which involve small uncertainty.

The intuition for the extreme case is straightforward. Let $A$ and $B$ represent the better alternatives in state $A$ and $B$ respectively. Suppose for the sufficient condition [3] that the prior probability is biased toward the state $B$. In the single person decision-making, the higher type decision-maker chooses the better alternative $A$ at the state $A$, because he has perfect information in the extreme case. The lower type decision-maker chooses the wrong alternative $B$ at the state $A$, because he receives less reliable information and the prior probability is sufficiently biased so that he ignores whatever signal he receives and chooses the alternative $B$. Hence, the single person decision-maker chooses the better alternative $A$ at the state $A$ if and only if he is the higher type. In the group decision-making by voting, the higher type member votes for $A$ and the lower type member votes for $B$ with high probabilities at the state $A$. This may lead to the choices opposite to the above described choices by the single person decision-making. First, a higher type member who would choose the alternative $A$ as the single person decision-maker votes for $A$ and this vote is overturned by other lower type members' votes for $B$. Second, a lower type member who would choose the alternative $B$ as the single person decision-maker votes for $B$ and this vote is overturned by other higher type members' votes for $A$. The first case is more likely than the second case, because a lower type is more likely than a higher type under the sufficient condition [2]. Hence, the committee chooses the better alternative $A$ at the state $A$ with lower probabilities under the group decision-making by voting than the single person decision-making. The intuition for the general cases is more complex, because if the prior probability distribution is sufficiently biased, a higher type member also chooses/votes for $B$ at the state $A$ so that the single person decision-making is superior to the group decision-making by voting under the prior probability distribution not extremely biased but sufficiently biased. However, the logic of the superiority is similar to the extreme case.

This result can be applied to the allocation problem of right to vote argued in the third part of the Condorcet jury theorem in a general setting. The possible allocations of right to vote are an allocation to one member, three members,..., $2m + 1$ members,..., all the members $(2n + 1)$. When the voting right is allocated to $2m + 1$ members ($m \geq 1$), the committee's decision is made by voting by the $2m+1$ members with the simple majority rule.[*1] We establish a sufficient condition under which the single person decision-making is the best procedure in the $n + 1$ possible allocations of right to vote.

The rest of this paper is as follows. In Section 2, we explain our model. In Section 3, we examine the optimal choice behavior in the single person decision-making and the equilibrium voting behavior in group decision-making by voting. In Section 4, we analyze the efficiency of decision. In Section 5, we compare the efficiency between the single person decision-making and the group decision-making by voting, and we present our main results. In Section 6, we apply the main result to establish a sufficient condition under which the single person decision-making is superior to the decision-making by voting by any $2m + 1$ members ($m \geq 1$) with the simple majority rule. Section 7 provides some concluding remarks.

[*1] To avoid the case of tie-breaking, we consider the case in which the right to vote are allocated to an odd number of members.

**Related Literature** Austen-Smith and Banks (1996) pointed out that the assumption of the classical Condorcet jury theorem may not hold under strategic voting. In their basic model, it is assumed that each member receives binary signal with the same degree of information precision. Realizing the issue of strategic voting, it has been studied since 1990s whether the Condorcet jury theorem holds or not under the strategic voting by analyzing the equilibrium of the voting game.[*2][*3]

The seminal work about the superiority of group decision-making by voting under the strategic voting is made by Wit (1998). Wit (1998) analyzed the basic model in which each member has the same degree of information precision. The key idea of Wit (1998) is that the equilibrium voting behavior maximizes the efficiency of decision, argued in McLenan (1998). Wit (1998) showed that the group can decide more efficiently than single person for any group size and any parameters in the model. We introduce the heterogeneity of the degree of information precision and the incomplete information about members' degrees of information precision. In contrast with Wit (1998), the superiority of group decision-making by voting does not hold for some parameter sets in our model.[*4]

Mukhopadhaya (2003) and Goertz (2014) showed that the superiority of group decision-making by voting does not hold in more complex models than Wit (1998). To be more specific, Mukhopadhaya (2003) considered costly information acquisition model in which each member chooses between acquiring costly information before his vote and voting without information. The group may not decide more efficiently than the single person because each member in the group has an incentive to free-ride on the other member's information acquisition. The group decision-making by voting may be inferior mainly due to its problem in the information acquisition rather than the information aggregation. In contrast, we showed the superiority of the single person decision-making in the information aggregation.

[*2] Gerling et al. (2005) and Li and Suen (2009) provide surveys of decision-making in committees and the Condorcet jury theorem.

[*3] The literature of the strategic voting paid attention to the asymptotic property of the Condorcet jury theorem. Feddersen and Pesendorfer (1998) showed that the asymptotic property of the Condorcet jury theorem holds under strategic voting for majority rules, except the unanimity rules. Duggan and Martinelli (2001) and Meirowitz (2002) extended the basic model to a model with the continuous type space. They showed that the asymptotic property of the Condorcet jury theorem holds for majority rules. Moreover, they showed that the asymptotic property of the Condorcet jury theorem holds for the unanimity rules if and only if the strong signal condition holds. Meirowitz (2002) also claimed that the superiority of group decision-making by voting may not hold for small $n$ while the superiority of group decision-making by voting holds for sufficiently large $n$ since the asymptotic property holds.

[*4] Feddersen and Pesendorfer (1996) and McMurray (2013) analyzed the strategic abstention model, in which each member is allowed to choose abstention. They showed that members who have lower degrees of information precision choose abstention and delegate the decision to members who have higher degrees of information precision. In their strategic abstention models, the superiority of group decision-making by voting holds because the members discard less reliable information in a form of abstention in their voting.

Goertz (2014) considered a model with three alternatives. The three-alternative model has many equilibria, and voting behaviors are distorted in one of the equilibria, in which each member does not vote for a particular alternative. There also exists efficient equilibrium in which each member vote informatively and the group decides more efficiently than the single person decision. The superiority of the single person decision-making in her model is derived by focusing on a particular inefficient equilibrium among multiple equilibria including the efficient one and relying on the particular nature of the equilibrium with the above distorted voting behavior. In contrast, we showed the superiority of the single person decision-making without resorting to the distortion of equilibrium behaviors of the above kind.

# 2 Model

We consider a $2n + 1$ member committee. The committee chooses between alternatives $A$ and $B$. The members have the same preference over the alternatives, depending on the state of the world. There are two states of the world, $\omega \in \{A, B\}$. We assume $\Pr(\omega = B) = q \geq 1/2$, that is, the common prior probability is biased for $B$. State $A$ is a state in which alternative $A$ is better than alternative $B$ for all the members. State $B$ is the opposite. We assume that the utility from choosing the better alternative is the same between state $A$ and $B$ and normalized to 1. We also assume that the utility from choosing the worse alternative is the same between state $A$ and $B$ and normalized to 0. Let $d \in \{A, B\}$ denote a committee's decision. Then, the utility function is as follows;

$$u(d = A|\omega = A) = u(d = B|\omega = B) = 1$$

and

$$u(d = A|\omega = B) = u(d = B|\omega = A) = 0.$$

Each member receives a signal that conveys information about the state of the world. The signal is realized in $S = \{a, b\}$. When the state is $\omega = A$, member $i$ receives signal $s_i = a$ with probability $t_i$ or $s_i = b$ with probability $1 - t_i$. Similarly, when the state is $\omega = B$, member $i$ receives signal $s_i = b$ with probability $t_i$ or $s_i = a$ with probability $1 - t_i$. The probability $t_i$ is interpreted as the degree of information precision of member $i$. We assume that the degree of information precision $t_i$ is private information of member $i$ and $t_i \in \{t_L, t_H\} = T$.[*5] The distribution of the degree of information precision is $\Pr(t_i = t_H) = p$. We assume that $1/2 < t_L < t_H \leq 1$ and we say that the strong signal condition holds if $t_H = 1$.[*6] Thus,

[*5] In the model of Wit (1998), it is assumed that each member's degree of information precision is the same.

[*6] The notion of strong signal condition is applied to the study of strategic voting with continuous signal model by Duggan and Martinelli (2001). The notion of strong signal here is interpreted as an application of their notion to the finite signal model.

each member has two kinds of private information, the signal $s_i$ and the degree of information precision $t_i$. The signal $s_i$ is realized independently across the members given the state. The degree of information precision $t_i$ is realized independently across the members and the states.

In this paper, we compare two alternative procedures for the committee's decision. One is the single person decision. A member is selected randomly and the committee delegates the committee's decision to him. The other is voting by the committee members. The voting rule is the simple majority rule. Each member votes for alternative $A$ or $B$.[*7] The committee's decision is $d = A$ if and only if at least $n + 1$ members vote for $A$.

The timing of the decision-making is as follows. In the single person decision, a member is selected randomly. Then, a state $\omega \in \{A, B\}$ is realized with prior probability $\Pr(\omega = B) = q \geq 1/2$. Each committee member $i$ is endowed with his information precision $t_i \in \{t_L, t_H\}$ with probability $\Pr(t_i = t_H) = p$. Then, each committee member $i$ receives a signal $s_i \in \{a, b\}$. The selected member chooses alternative between $A$ and $B$. His choice becomes the committee's decision, $d \in \{A, B\}$. In the procedure of voting by committee members, after the realization of the state, the degrees of information precision, and the signals, the committee members vote for alternative $A$ or $B$ simultaneously. A decision $d \in \{A, B\}$ is made by the simple majority rule. In both procedures, the members receives utilities depending on the decision made and the state realized.

# 3 Equilibrium Analysis

## 3.1 Choice Behavior in Single Person Decision-making

We consider the optimal choice behavior in the single person decision-making. Suppose that a member $i$ is selected to make the committee's decision. He will have private information $(s_i, t_i) \in S \times T$. Then, his choice strategy is a function $\sigma_i : S \times T \to [0, 1]$ where $\sigma_i(s, t)$ denotes the probability that he chooses alternative $A$ when he has $(s, t)$.

The condition by which a member who has $(s, t)$ weakly prefers alternative $A$ to $B$ is that the expected utility from choosing $A$ is more than or equal to the expected utility from choosing $B$. That is,

$$\begin{aligned}&\Pr(\omega = A|s, t) \times u(d = A|\omega = A) + \Pr(\omega = B|s, t) \times u(d = A|\omega = B)\\ &\geq \Pr(\omega = A|s, t) \times u(d = B|\omega = A) + \Pr(\omega = B|s, t) \times u(d = B|\omega = B).\end{aligned}$$

This condition is arranged as follows,

$$\frac{\Pr(\omega = A|s, t)}{\Pr(\omega = B|s, t)} \geq 1. \tag{1}$$

[*7]The members are not allowed to choose abstention.

In particular, the indifference condition is

$$\frac{\Pr(\omega = A|s,t)}{\Pr(\omega = B|s,t)} = 1. \tag{2}$$

Next Lemma shows that the ratio of posterior beliefs is monotone.

**Lemma 1.** *The ratio of posterior beliefs is*

$$\frac{\Pr(\omega = A|s,t)}{\Pr(\omega = B|s,t)} = \begin{cases} \frac{1-q}{q}\frac{t_H}{1-t_H} & \text{for } (s,t) = (a, t_H) \\ \frac{1-q}{q}\frac{t_L}{1-t_L} & \text{for } (s,t) = (a, t_L) \\ \frac{1-q}{q}\frac{1-t_L}{t_L} & \text{for } (s,t) = (b, t_L) \\ \frac{1-q}{q}\frac{1-t_H}{t_H} & \text{for } (s,t) = (b, t_H) \end{cases} \tag{3}$$

*and they are ordered as*

$$\frac{\Pr(\omega = A|a,t_H)}{\Pr(\omega = B|a,t_H)} > \frac{\Pr(\omega = A|a,t_L)}{\Pr(\omega = B|a,t_L)} > \frac{\Pr(\omega = A|b,t_L)}{\Pr(\omega = B|b,t_L)} > \frac{\Pr(\omega = A|b,t_H)}{\Pr(\omega = B|b,t_H)}. \tag{4}$$

*Proof.* The formula of (3) is immediately obtained by using the Bayes rule. The inequality (4) follows from (3) by $1/2 < t_L < t_H$. □

Therefore, the optimal choice behavior is as follows.

**Theorem 1.** *The optimal choice behavior is*

*(i)* $(\sigma_i(a,t_H), \sigma_i(a,t_L), \sigma_i(b,t_L), \sigma_i(b,t_H)) = (1,1,0,0)$ *for* $q \in [1/2, t_L)$,

*(ii)* $(\sigma_i(a,t_H), \sigma_i(a,t_L), \sigma_i(b,t_L), \sigma_i(b,t_H)) = (1,\sigma^*,0,0)$ *for* $q = t_L$,

*(iii)* $(\sigma_i(a,t_H), \sigma_i(a,t_L), \sigma_i(b,t_L), \sigma_i(b,t_H)) = (1,0,0,0)$ *for* $q \in (t_L, t_H)$,

*(iv)* $(\sigma_i(a,t_H), \sigma_i(a,t_L), \sigma_i(b,t_L), \sigma_i(b,t_H)) = (\sigma^{**},0,0,0)$ *for* $q = t_H$,

*(v)* $(\sigma_i(a,t_H), \sigma_i(a,t_L), \sigma_i(b,t_L), \sigma_i(b,t_H)) = (0,0,0,0)$ *for* $q \in (t_H, 1]$,

*where* $\sigma^*, \sigma^{**} \in [0,1]$.

*Proof.* Obvious by the indifference condition (2) and the inequality (4). □

The optimal choice behavior is weakly "monotone" in the prior probability $q$. Figure 1 illustrate the optimal choice behavior.

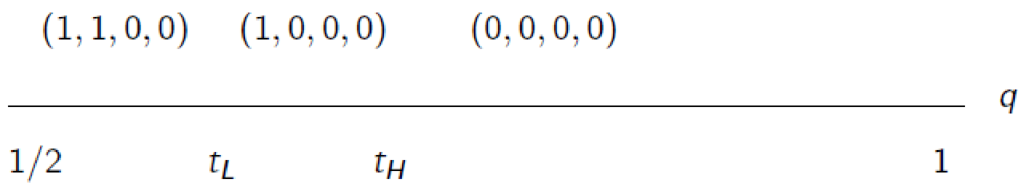


Figure 1: Optimal choice behavior

## 3.2 Voting Behavior in Group Decision-making

We consider the Bayesian equilibrium in the voting game. Each member $i$ has private information $(s_i, t_i) \in S \times T$. Then, member $i$'s voting strategy is a function $\sigma_i : S \times T \rightarrow [0, 1]$ where $\sigma_i(s_i, t_i)$ denotes the probability that he votes for alternative $A$ when he has $(s_i, t_i)$.

We focus on the equilibrium in symmetric strategy profiles. The condition by which a member who has $(s, t)$ weakly prefers voting for $A$ to $B$ is that the expected utility from voting for $A$ is more than or equal to the expected utility from voting for $B$. That is,

$$
\begin{aligned}
\Pr(\omega = A|s,t) \quad \times \quad & [\{\Pr(\text{more than } n+1 \text{ members vote for } A|\omega = A) \\
& + \Pr(n \text{ members vote for } A|\omega = A)\}u(d = A|\omega = A) \\
& + \{\Pr(\text{fewer than } n-1 \text{ members vote for } A|\omega = A)\}u(d = B|\omega = A)] \\
+ \Pr(\omega = B|s,t) \quad \times \quad & [\{\Pr(\text{more than } n+1 \text{ members vote for } A|\omega = B) \\
& + \Pr(n \text{ members vote for } A|\omega = B)\}u(d = A|\omega = B) \\
& + \{\Pr(\text{fewer than } n-1 \text{ members vote for } A|\omega = B)\}u(d = B|\omega = B)] \\
\geq \Pr(\omega = A|s,t) \quad \times \quad & [\{\Pr(\text{more than } n+1 \text{ members vote for } A|\omega = A)\}u(d = A|\omega = A) \\
& + \{\Pr(n \text{ members vote for } A|\omega = A) \\
& + \Pr(\text{fewer than } n-1 \text{ members vote for } A|\omega = A)\}u(d = B|\omega = A)] \\
+ \Pr(\omega = B|s,t) \quad \times \quad & [\{\Pr(\text{more than } n+1 \text{ members vote for } A|\omega = B)\}u(d = A|\omega = B) \\
& + \{\Pr(n \text{ members vote for } A|\omega = B) \\
& + \Pr(\text{fewer than } n-1 \text{ members vote for } A|\omega = B)\}u(d = B|\omega = B)].
\end{aligned} \tag{5}
$$

First, we consider symmetric equilibrium in unresponsive strategies. The member does not change his voting behavior according to his private information $(s, t)$ and votes for a particular alternative with probability one. Formally, the unresponsive strategies are

$$(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H)) = (0,0,0,0)$$

and

$$(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H)) = (1,1,1,1).$$

We can check that the unresponsive strategies always constitute the equilibrium.

**Lemma 2.** *The unresponsive strategies*

$$(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H)) = (0,0,0,0)$$

*and*

$$(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H)) = (1,1,1,1)$$

*constitute the equilibrium.*

*Proof.* We prove the case of $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H)) = (0,0,0,0)$. We consider member $i$'s best response to the above strategy. Under this strategy, other members vote for alternative $B$ with probability one in both states, $\omega = A, B$. Then, the member $i$'s expected utilities from voting for $A$ and from voting for $B$ with $(s,t)$ are

$$\begin{aligned}
&\Pr(\omega = A|s,t) \times [0 \times u(d = A|\omega = A) + 1 \times u(d = B|\omega = A)] \\
&+ \Pr(\omega = B|s,t) \times [0 \times u(d = A|\omega = B) + 1 \times u(d = B|\omega = B)] \\
&= \Pr(\omega = B|s,t)
\end{aligned}$$

and

$$\begin{aligned}
&\Pr(\omega = A|s,t) \times [0 \times u(d = A|\omega = A) + 1 \times u(d = B|\omega = A)] \\
&+ \Pr(\omega = B|s,t) \times [0 \times u(d = A|\omega = B) + 1 \times u(d = B|\omega = B)] \\
&= \Pr(\omega = B|s,t)
\end{aligned}$$

respectively. The member $i$ is indifferent between voting for $A$ and $B$ for any $(s,t)$. Then, the strategy $(0,0,0,0)$ is best response to $(0,0,0,0)$, and this strategy constitutes an equilibrium.

The case of $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H)) = (1,1,1,1)$ is established similarly. □

Next, we consider responsive strategies. Let

$$\gamma_A = \Pr(\text{a member votes for } A|\omega = A)$$

and

$$\gamma_B = \Pr(\text{a member votes for } B|\omega = B)$$

denote the voting probabilities that a member votes for the better alternative given the states. Under the responsive strategies, it holds that $0 < \gamma_A, \gamma_B < 1$. Let $piv$ denote an event in which a member is pivotal, that is, $n$ members vote for $A$ and the other $n$ members vote for $B$. Let

$$\Pr(piv|\omega = A, (\sigma_i(a,t_H), \sigma_i(a,t_L), \sigma_i(b,t_L), \sigma_i(b,t_H)))$$

and

$$\Pr(piv|\omega = B, (\sigma_i(a,t_H), \sigma_i(a,t_L), \sigma_i(b,t_L), \sigma_i(b,t_H)))$$

denote the probabilities that the member $i$ is pivotal given the states when the other members follow the strategy $(\sigma_i(a,t_H), \sigma_i(a,t_L), \sigma_i(b,t_L), \sigma_i(b,t_H))$. As long as there is no misunderstanding of the other members' strategy, we abbreviate them to $\Pr(piv|\omega = A)$ and $\Pr(piv|\omega = B)$.

Under the other members' responsive strategy, the condition (5) by which a member who has $(s,t)$ weakly prefers voting for $A$ to $B$ is arranged as

$$\frac{\Pr(\omega = A|s,t)}{\Pr(\omega = B|s,t)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} \geq 1, \tag{6}$$

where $\Pr(piv|\omega = B) \neq 0$ is guaranteed by $0 < \gamma_A, \gamma_B < 1$. In particular, the indifference condition is

$$\frac{\Pr(\omega = A|s,t)}{\Pr(\omega = B|s,t)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} = 1. \tag{7}$$

Remember that the first part of the left-hand side of (7), the ratio of posterior beliefs given $(s,t)$, satisfies the monotonicity of the ratio of posterior beliefs (4). This allows us to consider the following class of strategies.

**Definition 1** (monotone strategy)**.** *A strategy is a monotone strategy if*

1. $\sigma_i(a,t_H) \geq \sigma_i(a,t_L) \geq \sigma_i(b,t_L) \geq \sigma_i(b,t_H)$, *and*
2. *if there exists* $(s,t)$ *such that* $\sigma_i(s,t) \in (0,1)$ *then* $\sigma_i(s',t') \notin (0,1)$ *for all* $(s',t') \neq (s,t)$.

The monotone strategy means that the voting probability for $A$ becomes higher when his posterior belief for $A$ becomes higher.[*8] Moreover, there is at most one type mixes. The monotonicity of the ratio of the posterior beliefs implies that the best response is a monotone strategy if the other members use responsive strategies.

[*8] Note that the unresponsive strategies are also monotone strategies.

**Lemma 3.** *A best response to the other members' responsive strategies is a monotone strategy.*

*Proof.* The monotonicity of the ratio of posterior belief (4) implies that the best response defined by (6) and (7) must be a monotone strategy. □

By Lemma 3, we focus on the symmetric equilibrium in monotone strategies. If other members follow a symmetric monotone strategy, the second part of the left-hand side of (7), which represents the ratio of the probabilities that a member is pivotal, is also "monotone" in the other members voting behavior.

**Lemma 4** (pivotal monotonicity)**.** *The ratio of the probabilities that a member is pivotal,* $\frac{\Pr(piv|\omega=A)}{\Pr(piv|\omega=B)}$*, satisfies following properties.*

*(a) For* $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H))=(1,1,0,0)$*,* $\frac{\Pr(piv|\omega=A)}{\Pr(piv|\omega=B)}=1$*.*

*(b) For* $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H))=(1,\sigma,0,0)$*,* $\frac{\Pr(piv|\omega=A)}{\Pr(piv|\omega=B)}$ *is decreasing in* $\sigma$*.*

*(c) For* $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H))=(\sigma,0,0,0)$*,* $\frac{\Pr(piv|\omega=A)}{\Pr(piv|\omega=B)}$ *is decreasing in* $\sigma$*. Moreover,* $\frac{\Pr(piv|\omega=A)}{\Pr(piv|\omega=B)}\to\left[\frac{t_H}{1-t_H}\right]^n$ *as* $\sigma\to 0$*.*

*Proof.* The probability that the member is pivotal at state $\omega=A$ is

$$\Pr(piv|\omega=A)=\binom{2n}{n}\gamma_A^n(1-\gamma_A)^n$$

and the probability that the member is pivotal at state $\omega=B$ is

$$\Pr(piv|\omega=B)=\binom{2n}{n}\gamma_B^n(1-\gamma_B)^n.$$

Then, the ratio of probabilities that the member is pivotal is that

$$\frac{\Pr(piv|\omega=A)}{\Pr(piv|\omega=B)}=\left[\frac{\gamma_A}{(1-\gamma_B)}\frac{(1-\gamma_A)}{\gamma_B}\right]^n.$$

First, we prove $(a)$ of Lemma 4. Under the voting behavior

$$(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H))=(1,1,0,0),$$

the voting probabilities for better alternatives are

$$\gamma_A=\gamma_B=pt_H+(1-p)t_L(=E[t]).$$

Then,

$$\begin{aligned}\frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} &= \left[\frac{\gamma_A}{(1-\gamma_B)}\frac{(1-\gamma_A)}{\gamma_B}\right]^n \\ &= \left[\frac{E[t](1-E[t])}{(1-E[t])E[t]}\right]^n \\ &= 1.\end{aligned}$$

Second, we prove $(b)$ of Lemma 4. Under the voting behavior

$$(\sigma_i(a, t_H), \sigma_i(a, t_L), \sigma_i(b, t_L), \sigma_i(b, t_H)) = (1, \sigma, 0, 0),$$

the voting probabilities for better alternatives are

$$\gamma_A = pt_H + (1-p)t_L\sigma$$

and

$$\gamma_B = 1 - p(1-t_H) - (1-p)(1-t_L)\sigma.$$

We prove that both $\frac{\gamma_A}{1-\gamma_B}$ and $\frac{1-\gamma_A}{\gamma_B}$ are decreasing in $\sigma$. First,

$$\begin{aligned}\frac{\partial}{\partial\sigma}\left(\frac{\gamma_A}{1-\gamma_B}\right) &= \frac{1}{(1-\gamma_B)^2} \times [\gamma_A'(1-\gamma_B) + \gamma_A\gamma_B'] \\ &= \frac{1}{(1-\gamma_B)^2} \times [(1-p)t_L(p(1-t_H) + (1-p)(1-t_L)\sigma) \\ &\qquad -(1-p)(1-t_L)(pt_H + (1-p)t_L\sigma)] \\ &= \frac{(1-p)p}{(1-\gamma_B)^2} \times [t_L - t_H] \\ &< 0\end{aligned}$$

by $\gamma_A' = (1-p)t_L$ and $\gamma_B' = -(1-p)(1-t_L)$ and $t_L < t_H$. Similarly,

$$\begin{aligned}\frac{\partial}{\partial\sigma}\left(\frac{1-\gamma_A}{\gamma_B}\right) &= \frac{1}{\gamma_B^2} \times [-\gamma_A'\gamma_B - (1-\gamma_A)\gamma_B'] \\ &= \frac{1}{\gamma_B^2} \times [-(1-p)t_L(1-p(1-t_H) - (1-p)(1-t_L)\sigma) \\ &\qquad +(1-p)(1-t_L)(1-pt_H - (1-p)t_L\sigma)] \\ &= \frac{(1-p)}{\gamma_B^2} \times [-(2t_L - 1) - p(t_H - t_L)] \\ &< 0.\end{aligned}$$

Then, the ratio of the probabilities that the member is pivotal

$$\frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} = \left[\frac{\gamma_A}{(1-\gamma_B)} \frac{(1-\gamma_A)}{\gamma_B}\right]^n$$

is decreasing in $\sigma$ for $(1, \sigma, 0, 0)$.

Third, we prove $(c)$ of Lemma 4. Under the voting behavior

$$(\sigma_i(a, t_H), \sigma_i(a, t_L), \sigma_i(b, t_L), \sigma_i(b, t_H)) = (\sigma, 0, 0, 0),$$

the voting probabilities for better alternatives are

$$\gamma_A = pt_H\sigma$$

and

$$\gamma_B = 1 - p(1 - t_H)\sigma.$$

We prove that $\frac{1-\gamma_A}{\gamma_B} = \frac{1-pt_H\sigma}{1-p(1-t_H)\sigma}$ are decreasing in $\sigma$,

$$\begin{aligned}
\frac{\partial}{\partial\sigma}\left(\frac{1-\gamma_A}{\gamma_B}\right) &= \frac{1}{\gamma_B^2} \times [-\gamma_A'\gamma_B - (1-\gamma_A)\gamma_B'] \\
&= \frac{1}{\gamma_B^2} \times [-pt_H(1 - p(1-t_H)\sigma) + p(1-t_H)(1 - pt_H\sigma)] \\
&= \frac{p}{\gamma_B^2} \times [-(2t_H - 1)] \\
&< 0
\end{aligned}$$

by $\gamma_A' = pt_H$ and $\gamma_B' = -p(1 - t_H)$ and $t_H > 1/2$. Then, the ratio of the probabilities that the member is pivotal

$$\begin{aligned}
\frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} &= \left[\frac{\gamma_A}{(1-\gamma_B)} \frac{(1-\gamma_A)}{\gamma_B}\right]^n \\
&= \left[\frac{pt_H\sigma}{p(1-t_H)\sigma} \frac{1 - pt_H\sigma}{1 - p(1-t_H)\sigma}\right]^n \\
&= \left[\frac{t_H}{1-t_H} \frac{1 - pt_H\sigma}{1 - p(1-t_H)\sigma}\right]^n.
\end{aligned}$$

is decreasing in $\sigma$ for $(\sigma, 0, 0, 0)$. Finally, we can check that

$$\lim_{\sigma\to 0} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} = \left[\frac{t_H}{1-t_H}\right]^n$$

for $(\sigma, 0, 0, 0)$.

□

The intuition of Lemma 4 is as follows. When the member is pivotal, other $n$-members vote for $A$ and the other $n$-members vote for $B$. When $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H)) = (1,1,0,0)$, the voting probabilities for better alternatives $\gamma_A$ and $\gamma_B$ are the same. Then, the probabilities that the member is pivotal at each state, $\Pr(piv|\omega = A)$ and $\Pr(piv|\omega = B)$, are also the same. If the voting behavior is biased for $B$[*9], both of the probabilities of voting for $A$ at each states, $\gamma_A$ and $1-\gamma_B$, is smaller than the probabilities under $(1,1,0,0)$. Moreover, the voting probability for $A$ at state $B$ becomes smaller than the voting probability for $A$ at state $A$. Then, "one vote for $A$ and one vote for $B$" is more likely at state $A$. Since the event "pivotal" is "one vote for $A$ and one vote for $B$" with $n$-pairs, the probability that the member is pivotal at state $A$ is larger than at state $B$.

Lemma 1 and Lemma 4 establishes the next theorem.

**Theorem 2.** *We define $q_l, q_h, \bar{q}$ as*

$$\begin{aligned}
\frac{1-q_l}{q_l} &= \frac{1-t_L}{t_L}\left[\frac{p(1-t_H)}{pt_H}\frac{1-p(1-t_H)}{1-pt_H}\right]^n \\
\frac{1-q_h}{q_h} &= \frac{1-t_H}{t_H}\left[\frac{p(1-t_H)}{pt_H}\frac{1-p(1-t_H)}{1-pt_H}\right]^n \\
\frac{1-\bar{q}}{\bar{q}} &= \frac{1-t_H}{t_H}\left[\frac{1-t_H}{t_H}\right]^n.
\end{aligned}$$

*Then, $q_l < q_h < \bar{q}$, and there exists a symmetric equilibrium in responsive strategy for $q < \bar{q}$, and the voting strategy in the equilibrium is as follows;*

*(I):* $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H)) = (1,1,0,0)$ *for* $q \in [1/2, t_L)$,

*(II):* $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H)) = (1,\sigma_q,0,0)$ *for* $q \in [t_L, q_l]$,

*(III):* $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H)) = (1,0,0,0)$ *for* $q \in (q_l, q_h)$,

*(IV):* $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H)) = (\sigma_q,0,0,0)$ *for* $q \in [q_h, \bar{q}]$,

*(V):* $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H)) = (0,0,0,0)$ *for* $q \in (\bar{q}, 1]$,

*where $\sigma_q$ is unique and decreasing in $q$.*

*Proof.* First, the fact that $q_l < q_h < \bar{q}$ immediately follows from $1/2 < t_L < t_H$.

Second, we prove the existence of equilibrium for each case of $(I)$ through $(V)$.

[*9] That is, $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H))$ are $(1,\sigma,0,0)$, $(1,0,0,0)$, $(\sigma,0,0,0)$ or $(0,0,0,0)$.

$(I)$ : We consider the condition in which the member weakly prefers voting for $A$ to $B$ for the case of $q \in [1/2, t_L)$. When the voting behavior is $(1,1,0,0)$, the ratio of the probabilities that the member is pivotal satisfies

$$\frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} = 1$$

by Lemma 4. Then, under the strategy $(1,1,0,0)$, the left-hand side of (6) for $(a, t_L)$ satisfies

$$\frac{\Pr(\omega = A|a, t_L)}{\Pr(\omega = B|a, t_L)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} = \frac{1-q}{q} \frac{t_L}{1-t_L} > 1$$

and the left-hand side of (6) for $(b, t_L)$ satisfies

$$\frac{\Pr(\omega = A|b, t_L)}{\Pr(\omega = B|b, t_L)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} = \frac{1-q}{q} \frac{1-t_L}{t_L} < 1$$

since $1/2 < q < t_L$. Then, Lemma 1 implies

$$\begin{aligned} \frac{\Pr(\omega = A|a, t_H)}{\Pr(\omega = B|a, t_H)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} &> \frac{\Pr(\omega = A|a, t_L)}{\Pr(\omega = B|a, t_L)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} \\ &> 1 \\ &> \frac{\Pr(\omega = A|b, t_L)}{\Pr(\omega = B|b, t_L)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} \\ &> \frac{\Pr(\omega = A|b, t_H)}{\Pr(\omega = B|b, t_H)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)}. \end{aligned}$$

Therefore, $(1,1,0,0)$ constitutes an equilibrium.

$(II)$ : We consider the indifference condition (7) for the case of $q \in [t_L, q_l]$. When the voting behavior is $(1,1,0,0)$, it holds that

$$\begin{aligned} & \frac{\Pr(\omega = A|a, t_L)}{\Pr(\omega = B|a, t_L)} \frac{\Pr(piv|\omega = A, (1,1,0,0))}{\Pr(piv|\omega = B, (1,1,0,0))} \\ = & \left[ \frac{1-q}{q} \frac{t_L}{1-t_L} \right] \times 1 \\ \leq & \ 1 \end{aligned}$$

since $q \geq t_L$. When the voting behavior is $(1,0,0,0)$, it holds that

$$\begin{aligned}
&\frac{\Pr(\omega = A|a, t_L)}{\Pr(\omega = B|a, t_L)} \frac{\Pr(piv|\omega = A, (1,0,0,0))}{\Pr(piv|\omega = B, (1,0,0,0))} \\
= &\left[\frac{1-q}{q}\frac{t_L}{1-t_L}\right] \times \left[\frac{pt_H}{p(1-t_H)}\frac{1-pt_H}{1-p(1-t_H)}\right]^n \\
\geq &\ 1
\end{aligned}$$

since $q \leq q_l$. Lemma 4 implies that there exists unique $\sigma_q$ such that

$$\frac{\Pr(\omega = A|a, t_L)}{\Pr(\omega = B|a, t_L)} \frac{\Pr(piv|\omega = A, (1,\sigma_q,0,0))}{\Pr(piv|\omega = B, (1,\sigma_q,0,0))} = 1.$$

Therefore, by Lemma 1, the equilibrium voting behavior is $(1, \sigma_q, 0, 0)$. Moreover, $\sigma_q$ is decreasing in $q$ since the ratio of posterior beliefs

$$\frac{\Pr(\omega = A|a, t_L)}{\Pr(\omega = B|a, t_L)}$$

is strictly decreasing in $q$.

$(III)$ : We consider the condition in which the member weakly prefers voting for $A$ to $B$ for the case of $q \in (q_l, q_h)$. Under the strategy $(1,0,0,0)$, the left-hand side of (6) for $(a, t_L)$ satisfies

$$\begin{aligned}
&\frac{\Pr(\omega = A|a, t_L)}{\Pr(\omega = B|a, t_L)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} \\
= &\left[\frac{1-q}{q}\frac{t_L}{1-t_L}\right] \times \left[\frac{pt_H}{p(1-t_H)}\frac{1-pt_H}{1-p(1-t_H)}\right]^n \\
< &\ 1
\end{aligned}$$

and the left-hand side of (6) for $(a, t_H)$ satisfies

$$\begin{aligned}
&\frac{\Pr(\omega = A|a, t_H)}{\Pr(\omega = B|a, t_H)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} \\
= &\left[\frac{1-q}{q}\frac{t_H}{1-t_H}\right] \times \left[\frac{pt_H}{p(1-t_H)}\frac{1-pt_H}{1-p(1-t_H)}\right]^n \\
> &\ 1
\end{aligned}$$

since $q_l < q < q_h$. Then, Lemma 1 implies

$$\begin{aligned}
\frac{\Pr(\omega = A|a, t_H)}{\Pr(\omega = B|a, t_H)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} &> 1 \\
&> \frac{\Pr(\omega = A|a, t_L)}{\Pr(\omega = B|a, t_L)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} \\
&> \frac{\Pr(\omega = A|b, t_L)}{\Pr(\omega = B|b, t_L)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} \\
&> \frac{\Pr(\omega = A|b, t_H)}{\Pr(\omega = B|b, t_H)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)}.
\end{aligned}$$

Therefore, $(1, 0, 0, 0)$ constitutes an equilibrium.

$(IV)$ : We consider the indifference condition (7) for the case of $q \in [q_h, \bar{q}]$. When the voting behavior is $(1, 0, 0, 0)$, it holds that

$$\begin{aligned}
&\frac{\Pr(\omega = A|a, t_H)}{\Pr(\omega = B|a, t_H)} \frac{\Pr(piv|\omega = A, (1, 0, 0, 0))}{\Pr(piv|\omega = B, (1, 0, 0, 0))} \\
= &\left[\frac{1-q}{q} \frac{t_H}{1-t_H}\right] \times \left[\frac{pt_H}{p(1-t_H)} \frac{1-pt_H}{1-p(1-t_H)}\right]^n \\
\leq &\ 1
\end{aligned}$$

since $q \geq q_h$. When the voting behavior is $(0, 0, 0, 0)$, it holds that

$$\begin{aligned}
&\frac{\Pr(\omega = A|a, t_H)}{\Pr(\omega = B|a, t_H)} \frac{\Pr(piv|\omega = A, (0, 0, 0, 0))}{\Pr(piv|\omega = B, (0, 0, 0, 0))} \\
= &\left[\frac{1-q}{q} \frac{t_H}{1-t_H}\right] \times \left[\frac{pt_H}{p(1-t_H)}\right]^n \\
\geq &\ 1
\end{aligned}$$

since $q \leq \bar{q}$. Lemma 4 implies that there exists unique $\sigma_q$ such that

$$\frac{\Pr(\omega = A|a, t_H)}{\Pr(\omega = B|a, t_H)} \frac{\Pr(piv|\omega = A, (\sigma_q, 0, 0, 0))}{\Pr(piv|\omega = B, (\sigma_q, 0, 0, 0))} = 1.$$

Therefore, by Lemma 1, the equilibrium voting behavior is $(\sigma_q, 0, 0, 0)$. Moreover, $\sigma_q$ is decreasing in $q$ since the ratio of posterior beliefs

$$\frac{\Pr(\omega = A|a, t_H)}{\Pr(\omega = B|a, t_H)}$$

is strictly decreasing in $q$.

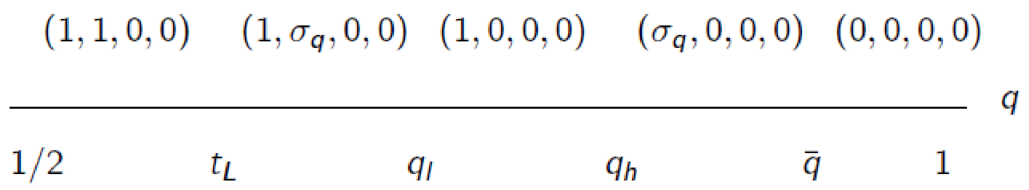


Figure 2: Equilibrium voting behavior

$(V)$ : We consider the condition in which the member weakly prefers voting for $A$ to $B$ for the case of $q \in (\bar{q}, 1]$. Under the strategy $(\sigma, 0, 0, 0)$, the left-hand side of (6) for $(a, t_H)$ satisfies

$$\begin{aligned} & \lim_{\sigma \to 0} \frac{\Pr(\omega = A|a, t_H)}{\Pr(\omega = B|a, t_H)} \frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} \\ = & \left[ \frac{1-q}{q} \frac{t_H}{1-t_H} \right] \times \left[ \frac{pt_H}{p(1-t_H)} \right]^n \\ < & \ 1 \end{aligned}$$

since $q > \bar{q}$. This means that the best response to $(\sigma, 0, 0, 0)$ is $(0, 0, 0, 0)$. Then, there does not exist symmetric equilibrium in responsive strategies. □

Theorem 2 indicates that the equilibrium voting behavior is weakly "monotone" in the prior probability $q$. Figure 2 illustrates the equilibrium voting behavior.

# 4 Efficiency Analysis

In this section, we consider the efficiency of single person decision-making and group decision-making respectively. The efficiency of decision is defined by the ex ante expected utility

$$\begin{aligned} E[u] &= \Pr(\omega = A) \left[ \Pr(d = A|\omega = A) u(d = A|\omega = A) + \Pr(d = B|\omega = A) u(d = B|\omega = A) \right] \\ &\quad + \Pr(\omega = B) \left[ \Pr(d = B|\omega = B) u(d = B|\omega = B) + \Pr(d = A|\omega = B) u(d = A|\omega = B) \right] \\ &= \Pr(\omega = A) \Pr(d = A|\omega = A) + \Pr(\omega = B) \Pr(d = B|\omega = B) \qquad (8) \end{aligned}$$

since each member has the same utility function.

## 4.1 Efficiency of Single Person Decision-making

We consider the efficiency of decision when the decision procedure is the single person decision-making. Let

$$\rho_A(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H))$$

and

$$\rho_B(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H))$$

denote the choice probabilities for better alternatives given the state when his choice behavior is $(\sigma_i(a,t_H),\sigma_i(a,t_L),\sigma_i(b,t_L),\sigma_i(b,t_H))$. Then, the efficiency of decision is

$$\begin{aligned} E_1[u] &= \Pr(\omega=A)\Pr(d=A|\omega=A)+\Pr(\omega=B)\Pr(d=B|\omega=B) \\ &= (1-q)\rho_A+q\rho_B. \end{aligned}$$

The efficiency of single person decision-making by optimal choice behavior stated in Theorem 1 is as follows.

**Theorem 3.** *The efficiency of decision when the decision procedure is the single person decision-making is*

$$E_1[u]=\begin{cases} pt_H+(1-p)t_L & \text{for } q\in[1/2,t_L] \\ pt_H+(1-p)q & \text{for } q\in(t_L,t_H) \\ q & \text{for } q\in[t_H,1] \end{cases}$$

*Proof.* We prove Theorem 3 for each case.

(i) For $q\in[1/2,t_L)$, the optimal choice is $(1,1,0,0)$. Then, the choice probabilities for better alternatives given the states are

$$\begin{aligned} \rho_A(1,1,0,0) &= pt_H+(1-p)t_L \\ &= E[t] \\ \rho_B(1,1,0,0) &= 1-p(1-t_H)-(1-p)(1-t_L) \\ &= E[t] \end{aligned}$$

and the efficiency of decision is

$$\begin{aligned} E_1[u](1,1,0,0) &= (1-q)E[t]+qE[t] \\ &= E[t]. \end{aligned}$$

(ii) For $q = t_L$, the optimal choice is $(1, \sigma^*, 0, 0)$ where $\sigma^* \in [0, 1]$. Then, the choice probabilities for better alternatives given the states are

$$\begin{aligned} \rho_A(1, \sigma^*, 0, 0) &= pt_H + (1-p)t_L\sigma^* \\ \rho_B(1, \sigma^*, 0, 0) &= 1 - p(1-t_H) - (1-p)(1-t_L)\sigma^* \end{aligned}$$

and the efficiency of decision is

$$\begin{aligned} E_1[u](1, \sigma^*, 0, 0) &= (1-q)[pt_H + (1-p)t_L\sigma^*] + q[1 - p(1-t_H) - (1-p)(1-t_L)\sigma^*] \\ &= pt_H + (1-p)q \\ &= pt_H + (1-p)t_L \\ &= E[t] \end{aligned}$$

by $t_L = q$.

(iii) For $q \in (t_L, t_H)$, the optimal choice is $(1, 0, 0, 0)$. Then, the choice probabilities for better alternatives given the states are

$$\begin{aligned} \rho_A(1, 0, 0, 0) &= pt_H \\ \rho_B(1, 0, 0, 0) &= 1 - p(1-t_H) \end{aligned}$$

and the efficiency of decision is

$$\begin{aligned} E_1[u](1, 0, 0, 0) &= (1-q)pt_H + q(1 - p(1-t_H)) \\ &= pt_H + (1-p)q. \end{aligned}$$

(iv) For $q = t_H$, the optimal choice is $(\sigma^{**}, 0, 0, 0)$ where $\sigma^{**} \in [0, 1]$. Then, the choice probabilities for better alternatives given the states are

$$\begin{aligned} \rho_A(\sigma^{**}, 0, 0, 0) &= pt_H\sigma^{**} \\ \rho_B(\sigma^{**}, 0, 0) &= 1 - p(1-t_H)\sigma^{**} \end{aligned}$$

and the efficiency of decision is

$$\begin{aligned} E_1[u](\sigma^{**}, 0, 0, 0) &= (1-q)[pt_H\sigma^{**}] + q[1 - p(1-t_H)\sigma^{**}] \\ &= q \end{aligned}$$

by $t_H = q$.

(v) For $q \in (t_H, 1]$, the optimal choice is $(0,0,0,0)$. Then, the choice probabilities for better alternatives given the states are

$$\begin{aligned}\rho_A(0,0,0,0) &= 0\\ \rho_B(0,0,0,0) &= 1\end{aligned}$$

and the efficiency of decision is

$$\begin{aligned}E_1[u](0,0,0,0) &= (1-q)0+q\\ &= q.\end{aligned}$$

□

Moreover, we establish the following result.

**Corollary 1** (convexity single)**.** *The efficiency of decision under optimal choice behavior when the decision procedure is the single person decision-making $E_1[u]$ is piece-wise linear, monotonically increasing and convex in $q$.*

Figure 3 illustrates the efficiency of single person decision-making. In Figure 3, we assume $p = 0.9$, $t_L = 0.6$ and $t_H = 0.9$.

## 4.2 Efficiency of Group Decision-making by Voting

We consider efficiency of decision when the decision procedure is voting with the simple majority rule. Let

$$\gamma_A(\sigma_i(a,t_H), \sigma_i(a,t_L), \sigma_i(b,t_L), \sigma_i(b,t_H))$$

and

$$\gamma_B(\sigma_i(a,t_H), \sigma_i(a,t_L), \sigma_i(b,t_L), \sigma_i(b,t_H))$$

denote the voting probabilities for better alternatives given the state when the symmetric voting behavior is $(\sigma_i(a,t_H), \sigma_i(a,t_L), \sigma_i(b,t_L), \sigma_i(b,t_H))$. Then, the probabilities that the committee chooses better alternatives given the state are

$$d_A = \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} \gamma_A^k (1-\gamma_A)^{2n+1-k} \tag{9}$$

and

$$d_B = \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} \gamma_B^k (1-\gamma_B)^{2n+1-k} \tag{10}$$

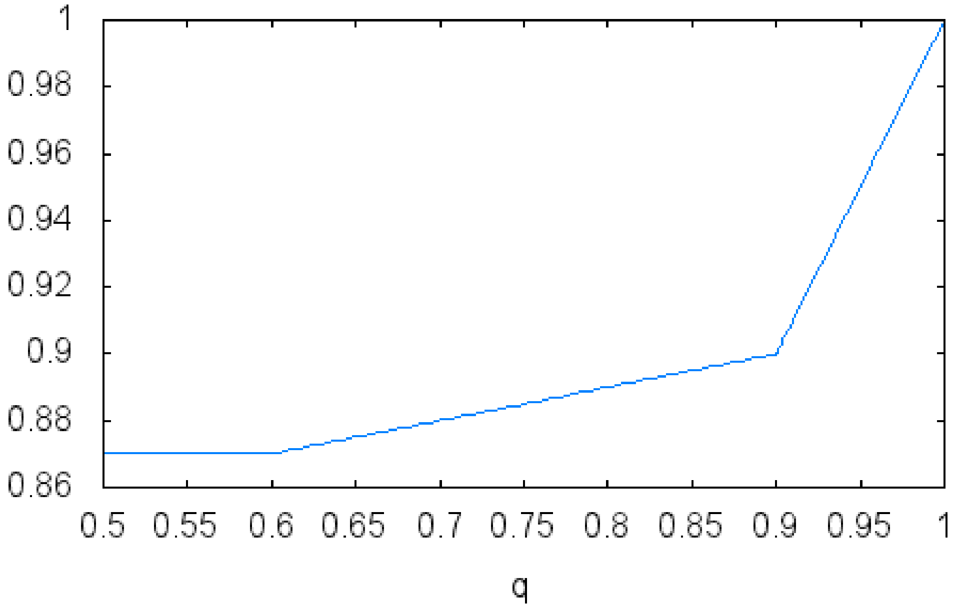


Figure 3: Efficiency of single person decision-making

respectively. The efficiency of decision is

$$\begin{aligned} E_{2n+1}[u] &= \Pr(\omega = A)\Pr(d = A|\omega = A) + \Pr(\omega = B)\Pr(d = B|\omega = B) \\ &= (1-q)d_A + qd_B. \end{aligned}$$

Next lemma shows that the voting strategy stated in Theorem 2 maximize the efficiency of decision in the class of symmetric strategies.[*10]

**Lemma 5.** *The equilibrium voting strategy maximizes the efficiency of decision in symmetric strategies.*

*Proof.* Consider any symmetric strategy

$$(\sigma_i(a, t_H), \sigma_i(a, t_L), \sigma_i(b, t_L), \sigma_i(b, t_H)) = (\sigma_1, \sigma_2, \sigma_3, \sigma_4).$$

Under this strategy, the voting probabilities for better alternatives are

$$\begin{aligned} \gamma_A &= pt_H\sigma_1 + (1-p)t_L\sigma_2 + (1-p)(1-t_L)\sigma_3 + p(1-t_H)\sigma_4 \\ \gamma_B &= 1 - p(1-t_H)\sigma_1 - (1-p)(1-t_L)\sigma_2 - (1-p)t_L\sigma_3 - pt_H\sigma_4. \end{aligned}$$

[*10]Wit (1998) proves a corresponding result with a model in which each member has an identical degree of information precision.

Note that

$$\begin{aligned}\frac{\partial d_A}{\partial \gamma_A} &= \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} \left[k\gamma_A^{k-1}(1-\gamma_A)^{2n+1-k} - (2n+1-k)\gamma_A^k(1-\gamma_A)^{2n-k}\right] \\ &= \binom{2n+1}{n+1}(n+1)\gamma_A^n(1-\gamma_A)^n\end{aligned}$$

because

$$-\binom{2n+1}{k}(2n+1-k)\gamma_A^k(1-\gamma_A)^{2n-k} + \binom{2n+1}{k+1}(k+1)\gamma_A^k(1-\gamma_A)^{2n+1-(k+1)} = 0$$

for $k = n+1, ..., 2n$. Similarly,

$$\begin{aligned}\frac{\partial d_B}{\partial \gamma_B} &= \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} \left[k\gamma_B^{k-1}(1-\gamma_B)^{2n+1-k} - (2n+1-k)\gamma_B^k(1-\gamma_B)^{2n-k}\right] \\ &= \binom{2n+1}{n+1}(n+1)\gamma_B^n(1-\gamma_B)^n.\end{aligned}$$

Then, we get

$$\begin{aligned}\frac{\partial}{\partial \sigma_1}E_{2n+1}[u] &= (1-q)\frac{\partial d_A}{\partial \gamma_A}\frac{\partial \gamma_A}{\partial \sigma_1} + q\frac{\partial d_B}{\partial \gamma_B}\frac{\partial \gamma_B}{\partial \sigma_1} \\ &= (1-q)\binom{2n+1}{n+1}(n+1)\gamma_A^n(1-\gamma_A)^n p t_H \\ &\quad +q\binom{2n+1}{n+1}(n+1)\gamma_B^n(1-\gamma_B)^n(-p(1-t_H)).\end{aligned}$$

Therefore, $\frac{\partial}{\partial \sigma_1}E_{2n+1}[u] \geq 0$ if and only if

$$\frac{1-q}{q}\frac{t_H}{1-t_H}\frac{\gamma_A^n(1-\gamma_A)^n}{\gamma_B^n(1-\gamma_B)^n} \geq 1,$$

which is equivalent to the condition (6),

$$\frac{\Pr(\omega = A|a, t_H)}{\Pr(\omega = B|a, t_H)}\frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} \geq 1,$$

by Lemma 1 and Lemma 4. Similarly,

$$
\begin{aligned}
& \frac{\partial}{\partial \sigma_2} E_{2n+1}[u] = (1-q)\frac{\partial d_A}{\partial \gamma_A}\frac{\partial \gamma_A}{\partial \sigma_2} + q\frac{\partial d_B}{\partial \gamma_B}\frac{\partial \gamma_B}{\partial \sigma_2} \geq 0 \\
\Leftrightarrow\ & \frac{1-q}{q}\frac{t_L}{1-t_L}\frac{\gamma_A^n(1-\gamma_A)^n}{\gamma_B^n(1-\gamma_B)^n} \geq 1 \\
\Leftrightarrow\ & \frac{\Pr(\omega = A|a, t_L)}{\Pr(\omega = B|a, t_L)}\frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} \geq 1,
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\partial}{\partial \sigma_3} E_{2n+1}[u] = (1-q)\frac{\partial d_A}{\partial \gamma_A}\frac{\partial \gamma_A}{\partial \sigma_3} + q\frac{\partial d_B}{\partial \gamma_B}\frac{\partial \gamma_B}{\partial \sigma_3} \geq 0 \\
\Leftrightarrow\ & \frac{1-q}{q}\frac{1-t_L}{t_L}\frac{\gamma_A^n(1-\gamma_A)^n}{\gamma_B^n(1-\gamma_B)^n} \geq 1 \\
\Leftrightarrow\ & \frac{\Pr(\omega = A|b, t_L)}{\Pr(\omega = B|b, t_L)}\frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} \geq 1,
\end{aligned}
$$

and

$$
\begin{aligned}
& \frac{\partial}{\partial \sigma_4} E_{2n+1}[u] = (1-q)\frac{\partial d_A}{\partial \gamma_A}\frac{\partial \gamma_A}{\partial \sigma_4} + q\frac{\partial d_B}{\partial \gamma_B}\frac{\partial \gamma_B}{\partial \sigma_4} \geq 0 \\
\Leftrightarrow\ & \frac{1-q}{q}\frac{1-t_H}{t_H}\frac{\gamma_A^n(1-\gamma_A)^n}{\gamma_B^n(1-\gamma_B)^n} \geq 1 \\
\Leftrightarrow\ & \frac{\Pr(\omega = A|b, t_H)}{\Pr(\omega = B|b, t_H)}\frac{\Pr(piv|\omega = A)}{\Pr(piv|\omega = B)} \geq 1.
\end{aligned}
$$

Therefore, the equilibrium voting behavior maximizes the efficiency of decision in symmetric strategies. □

Using Lemma 5, we can show that the efficiency of group decision-making by equilibrium voting behavior stated in Theorem 2 satisfies the following properties.

**Theorem 4.** *The efficiency of decision in the equilibrium when the decision procedure is voting with the simple majority rule satisfies the following properties.*

*(I) For* $q \in [1/2, t_L)$,

$$
E_{2n+1}[u] = \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} (E[t])^k (1-E[t])^{2n+1-k}.
$$

*(II) For $q \in [t_L, q_l]$, $E_{2n+1}[u]$ is monotonically increasing and convex in $q$.*

*(III) For $q \in (q_l, q_h)$, $E_{2n+1}[u]$ is monotonically increasing and linear in $q$.*

*(IV) For $q \in [q_h, \bar{q}]$, $E_{2n+1}[u]$ is monotonically increasing and convex in $q$.*

*(V) For $q \in (\bar{q}, 1]$,*

$$E_{2n+1}[u] = q.$$

*Proof.* First, we prove the cases of $(I)$ and $(V)$.

- For $q \in [1/2, t_L)$, the optimal voting behavior is $(1, 1, 0, 0)$. Then, the voting probabilities for better alternatives given the states are

$$\begin{aligned}
\gamma_A(1,1,0,0) &= pt_H + (1-p)t_L \\
&= E[t] \\
\gamma_B(1,1,0,0) &= 1 - p(1-t_H) - (1-p)(1-t_L) \\
&= pt_H + (1-p)t_L \\
&= E[t].
\end{aligned}$$

  Then, the decision probabilities are

$$d_A = d_B = \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} (E[t])^k (1-E[t])^{2n+1-k}.$$

  Therefore, the efficiency of decision is

$$E_{2n+1}[u](1,1,0,0) = \sum_{k=n+1}^{2n+1} \binom{2n+1}{k} (E[t])^k (1-E[t])^{2n+1-k}.$$

- For $q \in (\bar{q}, 1]$, the optimal voting behavior is $(0, 0, 0, 0)$. Then, the voting probabilities for better alternatives given the states are

$$\begin{aligned}
\gamma_A(0,0,0,0) &= 0 \\
\gamma_B(0,0,0,0) &= 1.
\end{aligned}$$

  Then, the decision probabilities are

$$d_A = 0$$

and

$$d_B = 1.$$

Therefore, the efficiency of decision is

$$\begin{aligned} E_{2n+1}[u](0,0,0,0) &= (1-q)0 + q \\ &= q. \end{aligned}$$

Second, we prove that $E_{2n+1}[u]$ is monotonically increasing in $q$ for the cases $(II)$, $(III)$ and $(IV)$. By Lemma 5, we can apply the envelop theorem, and then we have

$$\begin{aligned} \frac{\partial E_{2n+1}[u]}{\partial q} &= -d_A + d_B \\ &> 0, \end{aligned}$$

because $\gamma_A < \gamma_B$ holds under the strategies stated in Theorem 2 for $(II)$, $(III)$ and $(IV)$, and $\gamma_A < \gamma_B$ implies $d_A < d_B$ by the fact that

$$\frac{\partial d}{\partial \gamma} = \binom{2n+1}{n+1}(n+1)\gamma^n(1-\gamma)^n > 0,$$

which we derived in the proof of Lemma 5. Hence, $E_{2n+1}[u]$ is increasing in $q$.

Third, we prove that $E_{2n+1}[u]$ is convex in $q$ for the cases of $(II)$ and $(IV)$. For the case of $(II)$, the equilibrium voting behavior is $(1, \sigma, 0, 0)$ and it holds that

$$\begin{aligned} &\frac{\partial^2 E_{2n+1}[u]}{\partial q^2} \\ &= \left(-\frac{\partial d_A}{\partial \gamma_A}\frac{\partial \gamma_A}{\partial \sigma} + \frac{\partial d_B}{\partial \gamma_B}\frac{\partial \gamma_B}{\partial \sigma}\right)\frac{\partial \sigma}{\partial q} \\ &= \left(-\frac{(2n+1)!}{n!n!}\gamma_A^n(1-\gamma_A)^n(1-p)t_L - \frac{(2n+1)!}{n!n!}\gamma_B^n(1-\gamma_B)^n(1-p)(1-t_L)\right)\frac{\partial \sigma}{\partial q} \\ &> 0 \end{aligned}$$

because $\frac{\partial \sigma}{\partial q} < 0$ by Theorem 2. Hence, $E_{2n+1}[u]$ is convex in $q$ for the case $(II)$. A similar argument holds for the case $(IV)$.

Finally, we prove that $E_{2n+1}[u]$ is linear in $q$ for the case $(III)$. For the case $(III)$, the equilibrium voting behavior is $(1, 0, 0, 0)$. Then, the voting probabilities for better alternatives is independent of $q$. Therefore, the probabilities that the committee chooses better alternatives, $d_A$ and $d_B$, are also independent of $q$. Hence,

$$E_{2n+1}[u](1,0,0,0) = (1-q)d_A + q d_B$$

is linear in $q$. □

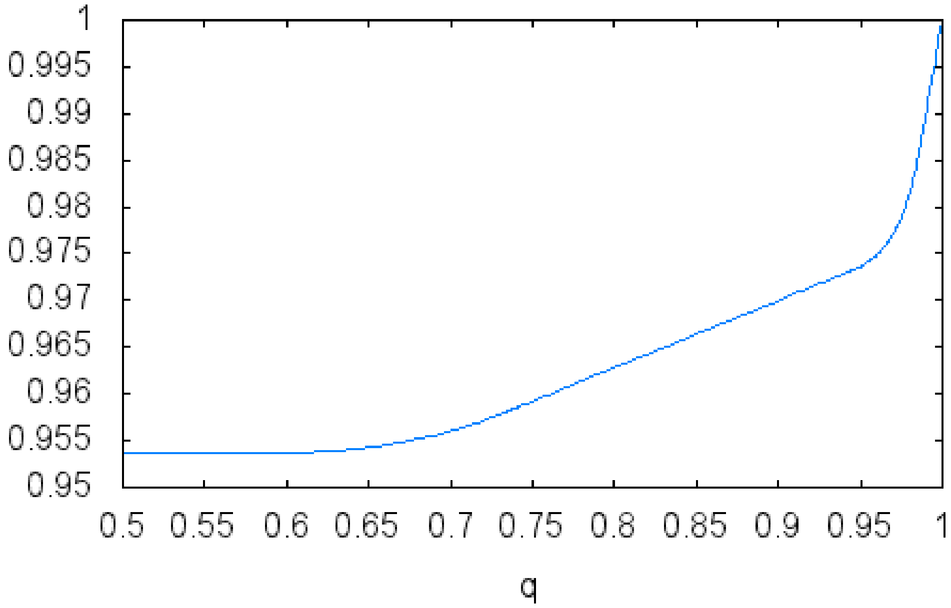


Figure 4: Efficiency of group decision-making

Figure 4 illustrates the efficiency of group decision-making by voting by three-members. In Figure 4, we assume $p = 0.9$, $t_L = 0.6$ and $t_H = 0.9$. Then, $q_l = 513/695(\approx 0.7381)$, $q_h = 1539/1630(\approx 0.9441)$ and $\bar{q} = 81/82(\approx 0.9878)$.

# 5 Comparison of Efficiency

We compare the efficiency of decision between single person decision-making and group decision-making by voting. We define

$$\Delta = E_{2n+1}[u](\gamma_A, \gamma_B) - E_1[u](\rho_A, \rho_B) \tag{11}$$

where $\gamma_A$ and $\gamma_B$ are the voting probabilities for better alternative in equilibrium under group decision-making by voting and $\rho_A$ and $\rho_B$ are the optimal choice probabilities under single person decision-making. We say that the group decision-making is more efficient than single person decision-making if

$$\Delta \geq 0, \tag{12}$$

and we say that the single person decision-making is more efficient than group decision-making if

$$\Delta < 0. \tag{13}$$

In our model, the parameters are prior probability $q$, probability distribution of the degree of information precision $p$, and the degree of information precision $t_H$ and $t_L$. In this section, we examine the conditions on these parameters under which $\Delta \geq 0$ or $\Delta < 0$ holds.

## 5.1 Condorcet Jury Theorem under Strategic Voting

First, we provide a sufficient condition under which group decision-making is more efficient. We establish the following result.[*11]

**Theorem 5.** *For $q \notin (t_L, \min\{q_l, t_H\})$, it holds that $\Delta(q) \geq 0$. Moreover, $\Delta(q) = 0$ holds only if $q \geq \bar{q}$.*

*Proof.* First, we prove the case of $q \in [1/2, t_L]$. For $q \in [1/2, t_L]$, the optimal choice behavior in single person decision-making is $(1, 1, 0, 0)$ by Theorem 1. Then, $\rho_A = \rho_B = E[t]$. On the other hand, the equilibrium voting behavior is also $(1, 1, 0, 0)$ by Theorem 2. Therefore, $\gamma_A = \rho_A$ and $\gamma_B = \rho_B$. Note that $E[t] > 1/2$ since $1/2 < t_L < t_H$. Hence, $d_A > \rho_A$ and $d_B > \rho_B$ by the classical Condorcet jury theorem. Then, $\Delta(q) > 0$ for $q \in [1/2, t_L]$.

Second, we prove the case of $q \in [\bar{q}, 1]$. For $q \in [\bar{q}, 1]$, the optimal choice behavior in single person decision-making is $(0, 0, 0, 0)$ by Theorem 1 because $\bar{q} > t_H$. Then, $\rho_A = 0$ and $\rho_B = 1$. On the other hand, the equilibrium voting behavior is also $(0, 0, 0, 0)$ by Theorem 2. Therefore, $\gamma_A = 0$ and $\gamma_B = 1$. Then, $d_A = 0$ and $d_B = 1$. Hence, $\Delta(q) = 0$ for $q \in [\bar{q}, 1]$.

Finally, we prove the case of $q \in (\min\{q_l, t_H\}, \bar{q})$.

- Suppose $\min\{q_l, t_H\} = t_H$ and consider the case of $q \in (t_H, \bar{q})$. The optimal choice behavior in single person decision-making for $q \in (t_H, \bar{q})$ is $(0, 0, 0, 0)$. On the other hand, the equilibrium voting behavior for $q \in (t_H, \bar{q})$ is $(1, \sigma_q, 0, 0)$, $(1, 0, 0, 0)$, or $(\sigma_q, 0, 0, 0) \neq (0, 0, 0, 0)$. Then, $E_{2n+1}[u^*] > E_{2n+1}[u](0, 0, 0, 0)$ because the equilibrium voting behavior is the unique maximizer of the efficiency of decision by Lemma 5. Hence, $E_{2n+1}[u](0, 0, 0, 0) = E_1[u](0, 0, 0, 0)$ implies $E_{2n+1}[u^*] > E_1[u](0, 0, 0, 0)$.

- Suppose $\min\{q_l, t_H\} = q_l$ and consider the case of $q \in (q_l, \bar{q})$. The optimal choice behavior in single person decision-making for $q \in (q_l, \bar{q})$ is $(1, 0, 0, 0)$ or $(0, 0, 0, 0)$. On the other hand, the equilibrium voting behavior for $q \in (q_l, \bar{q})$ is $(1, 0, 0, 0)$ or $(\sigma_q, 0, 0, 0)$. The

[*11] Wit (1998) proves a similar result with a model in which each member has an identical degree of information precision. Our proof is parallel to the proof of Wit (1998).

proof of the case in which the optimal choice behavior in single person decision-making is $(0,0,0,0)$ is the same as the previous case of $\min\{q_l, t_H\} = t_H$.

We prove the case in which the optimal choice behavior in single person decision-making is $(1,0,0,0)$. The optimal choice behavior is $(1,0,0,0)$ when $q \in (q_l, t_H)$. The equilibrium voting behavior for $q \in (q_l, t_H)$ is also $(1,0,0,0)$ because $t_H < q_h$. Therefore, it holds that $\gamma_A = \rho_A$ and $\gamma_B = \rho_B$ for $q \in (q_l, t_H)$. Now, we consider a committee with $2k+1$ members for $k < n$. Let $q_l(k)$ and $q_h(k)$ be the thresholds for which $(1,0,0,0)$ is the equilibrium voting behavior with $2k+1$ members. Then, $q_l(k) < q_l$ because $q_l(k)$ is increasing in $k$ by the construction in Theorem 2. It also holds that $t_H < q_h(k)$ by the construction in Theorem 2. Hence, the equilibrium voting behavior with $2k+1$ members for $q \in (q_l, t_H)$ is also $(1,0,0,0)$ for $k < n$. This means that $\gamma_A$ and $\gamma_B$ also represent the voting probabilities for better alternative in $2k+1$ members committee. The difference of the efficiency of decision in equilibrium between $2k+1$ and $2k-1$ is

$$\begin{aligned}
\Delta_{2k-1}^{2k+1}(q) &\equiv [(1-q)d_{A,2k+1}(\gamma_A) + qd_{B,2k+1}(\gamma_B)] \\
&\quad -[(1-q)d_{A,2k-1}(\gamma_A) + qd_{B,2k-1}(\gamma_B)] \\
&= (1-q)[d_{A,2k+1}(\gamma_A) - d_{A,2k-1}(\gamma_A)] \\
&\qquad +q[d_{B,2k+1}(\gamma_B) - d_{B,2k-1}(\gamma_B)] \\
&= (1-q)\left[\binom{2k-1}{k}\gamma_A^k(1-\gamma_A)^k(2\gamma_A - 1)\right] \\
&\qquad +q\left[\binom{2k-1}{k}\gamma_B^k(1-\gamma_B)^k(2\gamma_B - 1)\right] \\
&= \binom{2k-1}{k} \times \left\{(1-q)\gamma_A^k(1-\gamma_A)^k(2\gamma_A-1)\right. \\
&\qquad \left. -q\,\gamma_B^k(1-\gamma_B)^k(2(1-\gamma_B)-1)\right\} \\
&= \binom{2k-1}{k} \times \left\{(1-q)\gamma_A^k(1-\gamma_A)^k 2\gamma_A - q\gamma_B^k(1-\gamma_B)^k 2(1-\gamma_B)\right. \\
&\qquad \left. -(1-q)\gamma_A^k(1-\gamma_A)^k + q\gamma_B^k(1-\gamma_B)^k\right\}.
\end{aligned}$$

We can check that $\Delta_{2k-1}^{2k+1} > 0$ for any $k \leq n$, because

$$\begin{aligned}
\frac{1-q}{q}\frac{2\gamma_A}{2(1-\gamma_B)}\left[\frac{\gamma_A(1-\gamma_A)}{\gamma_B(1-\gamma_B)}\right]^k &= \frac{1-q}{q}\frac{t_H}{1-t_H}\left[\frac{\gamma_A(1-\gamma_A)}{\gamma_B(1-\gamma_B)}\right]^k \\
&> 1
\end{aligned}$$

and

$$\frac{1-q}{q}\left[\frac{\gamma_A(1-\gamma_A)}{\gamma_B(1-\gamma_B)}\right]^k < \frac{1-q}{q}\frac{t_L}{1-t_L}\left[\frac{\gamma_A(1-\gamma_A)}{\gamma_B(1-\gamma_B)}\right]^k$$
$$< 1$$

hold by the equilibrium condition. Hence, $\Delta(q) = \Delta_1^{2n+1}(q) > 0$ for $q \in (q_l, t_H)$.

Thus, $\Delta(q) > 0$ for $q \in (\min\{q_l, t_H\}, \bar{q})$ for both cases of $\min\{q_l, t_H\}, \bar{q}) = t_H$ and $\min\{q_l, t_H\}, \bar{q}) = q_l$.

Therefore, $\Delta(q) \geq 0$ for $q \notin (t_L, \min\{q_l, t_H\})$ and $\Delta(q) = 0$ only if $q \geq \bar{q}$. □

The intuition of Theorem 5 is as follows. We decompose the difference $\Delta$ of the efficiency of decision between the group decision-making by voting and the single person decision-making, defined by equation (11), into two parts; the first is the difference of the efficiency when the voting behavior is assumed to be the same as the optimal choice behavior in the single parson decision-making, and the second is the difference of the efficiency of group decision-making by voting between the above assumed voting behavior and the equilibrium voting behavior. We can apply the Condorcet's argument to the first part and the Wit (1998)'s argument to the second part in the case considered in Theorem 5. In the second part, the equilibrium voting behavior is no worse than the assumed voting behavior. Then, it is enough to check the first part.

Specifically, the optimal choice behavior of the single person decision-making is (1,1,0,0) or (0,0,0,0) in the case of $q \notin (t_L, \min\{q_l, t_H\})$.[*12] In the former choice behavior, the decision-maker chooses an alternative $A$ when he receives signal $s = a$ and an alternative $B$ when he receives signal $s = b$. This behavior is called the informative choice behavior. In the latter choice behavior, the decision-maker ignores the signal and always chooses an alternative $B$. This behavior is called the unresponsive choice behavior.

1. When the informative choice behavior is optimal, the probability that the decision-maker chooses the better alternative at a state is larger than 1/2 at both of the states $A$ and $B$. If the group member takes the same behavior in the group decision-making by voting, the probability that the committee chooses the better alternative at a state is improved at both of the states $A$ and $B$, by the Condorcet jury theorem. Then, the efficiency of group decision-making by voting is strictly higher than the efficiency of single person decision-making in the first part comparison.

[*12] If $\min\{q_l, t_H\} = q_l$, the optimal choice behavior is (1,0,0,0) for $q \in (q_l, t_H)$. In this case, we need a more elaborated argument.

2. When the unresponsive choice behavior is optimal, the probability that the decision-maker chooses the better alternative $A$ at state $A$ is 0 and the probability that the decision-maker chooses the better alternative $B$ at state $B$ is 1. If the group member takes the same behavior in the group decision-making by voting, the probability that the committee chooses the better alternative $A$ at state $A$ is 0 and the probability that the committee chooses the better alternative $B$ at state $B$ is 1. Then, the efficiency of group decision-making by voting is the same as the efficiency of single person decision-making in the first part comparison.

This intuition is the same as that of Wit (1998). In his model, the degree of information precision is only one type. This implies that the optimal choice behavior in the single person decision-making in his model is either the informative choice behavior or the unresponsive choice behavior, over the whole range of $q \in (1/2, 1)$. In contrast, an intermediate choice behavior between the informative choice behavior and the unresponsive choice behavior is optimal in the single person decision-making for $q \in (t_L, t_H)$ in our model, in which there are two types of the degree of information precision. Specifically, the decision-maker who has $t = t_H$ chooses an alternative based on his signal while the decision-maker who has $t = t_L$ chooses the alternative $B$ irrespective of his signal. Under this behavior, the probability that the decision-maker chooses the better alternative may be less than 1/2, depending on the parameters $p$ and $t_H$. Then, the first part comparison is reversed. If the reversed first part comparison dominates the second part comparison, the single person decision-making is more efficient than the group decision-making by voting. Below, we examine this possibility of the superiority of the single person decision-making.

## 5.2 Reverse-Condorcet Jury Theorem under Strategic Voting

In this section, we pursue a reversed version of the Condorcet jury theorem and examine sufficient conditions under which the single person decision-making is more efficient than the group decision-making by voting. From the discussion after Theorem 5 in the previous section, the case we consider is $q \in (t_L, \min\{q_l, t_H\})$. In this case, the equilibrium voting behavior is $(1, \sigma_q, 0, 0)$ and the optimal choice behavior in single person decision-making is $(1, 0, 0, 0)$.

First, we analyze the case in which the storing signal condition holds. Then, we analyze the case in which the strong signal condition does not hold.

### 5.2.1 The Case with Strong Signal Condition

Under the strong signal condition $t_H = 1$, it holds that $q_l = 1$. The equilibrium voting behavior converges to $(1, 0, 0, 0)$ as $q$ goes to 1. Then, $\gamma_A \to p$ and $\gamma_B \to 1$. From this fact, we establish the following result.

**Theorem 6.** *If $p \geq 1/2$, $\Delta(q) \geq 0$ for $q \in (t_L, 1)$. If $p < 1/2$, there exist $\tilde{q} \in (t_L, 1)$ such that $\Delta(q) \geq 0$ for $q \in (t_L, \tilde{q}]$ and $\Delta(q) < 0$ for $q \in (\tilde{q}, 1)$.*

*Proof.* By the envelop theorem,

$$\Delta'(q) = -[d_A - \rho_A] + [d_B - \rho_B].$$

Then,

$$\lim_{q \to 1} \Delta'(q) = -[d_A(p) - p]$$

since $\gamma_A \to p$, $\rho_A \to p$, $\gamma_B \to 1$, and $\rho_B \to 1$ as $q \to 1$. By the classical Condorcet jury theorem, $d_A(p) - p > 0$ if and only if $p > 1/2$.

Note that $\Delta(q)$ is convex in $q \in (t_L, 1)$ by Corollary 1 and Theorem 4. Note also that $\Delta(t_L) > 0$ and $\Delta(1) = 0$ by Theorem 3 and Theorem 4. Therefore, there exists unique $\tilde{q} \in (t_L, 1)$ with $\Delta(\tilde{q}) = 0$ if and only if $p < 1/2$. This means that if $p \geq 1/2$, $\Delta(q) \geq 0$ for $q \in (t_L, 1)$. If $p < 1/2$, $\Delta(q) \geq 0$ for $q \in (t_L, \tilde{q}]$ and $\Delta(q) < 0$ for $q \in (\tilde{q}, 1)$. □

Combining Theorem 5 and Theorem 6, we have the following characterization of comparison of efficiency under the strong signal condition.

**Proposition 1.** *Suppose the strong signal condition holds. Then, if $p \geq 1/2$, $\Delta(q) \geq 0$ for $q \in [1/2, 1)$. If $p < 1/2$, there exist $\tilde{q} \in (t_L, 1)$ such that $\Delta(q) \geq 0$ for $q \in [1/2, \tilde{q}]$ and $\Delta(q) < 0$ for $q \in (\tilde{q}, 1)$.*

Figure 5 illustrates Corollary 1. In Figure 5, we compare the efficiency of decision between the group decision-making by three members and the single person decision-making. We assume that $p = 0.1$, $t_L = 0.6$ and $t_H = 1$. Then, the optimal choice behavior is $(1, 1, 0, 0)$ for $q < 0.6$ and $(1, 0, 0, 0)$ for $q > 0.6$. The equilibrium voting behavior is $(1, 1, 0, 0)$ for $q < 0.6$ and $(1, \sigma_q, 0, 0)$ for $q > 0.6$.

### 5.2.2 The Case without Strong Signal Condition

We consider the remaining case in which the strong signal condition does not hold, $t_H < 1$. First, we establish a sufficient condition under which the group decision-making is more efficient than the single person decision-making.

**Theorem 7.** *If $pt_H \geq 1/2$, it holds that $\Delta(q) > 0$ for $q \in (t_L, \min\{q_l, t_H\})$.*

*Proof.* For $q \in (t_L, \min\{q_l, t_H\})$, the optimal choice behavior in single person decision-making is $(1, 0, 0, 0)$. Since $\rho_A = pt_H \geq 1/2$ and $\rho_B = 1 - p(1 - t_H) > 1/2$, it holds that

$$E_{2n+1}[u](1, 0, 0, 0) > E_1[u](1, 0, 0, 0)$$

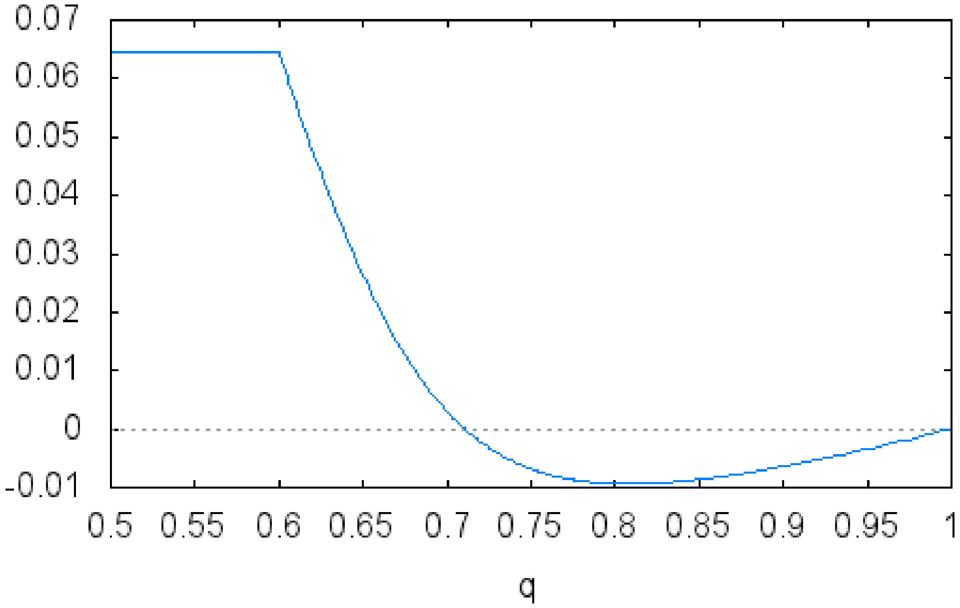


Figure 5: The difference of efficiency with the strong signal condition

by the classical Condorcet jury theorem.

On the other hand, the equilibrium voting behavior is $(1, \sigma_q, 0, 0)$ for $q \in (t_L, \min\{q_l, t_H\})$, and Lemma 5 implies

$$E_{2n+1}[u](1, \sigma_q, 0, 0) > E_{2n+1}[u](1, 0, 0, 0)$$

for $q \in (t_L, \min\{q_l, t_H\})$. Then, $\Delta(q) > 0$ for any $q$ if $pt_H > 1/2$. □

Combining Theorem 5 and Theorem 7, we have the following.

**Proposition 2.** *Suppose that the strong signal condition does not hold and* $pt_H \geq 1/2$. *Then,* $\Delta(q) \geq 0$ *for* $q \in [1/2, 1)$.

In the rest of this section, we assume that $pt_H < 1/2$. In Lemma 5, we stated that the efficiency of group decision-making in equilibrium maximizes the efficiency of group decision-making in symmetric strategies. Then, the efficiency of group decision-making in equilibrium is continuous for parameters because we can apply the Berge's maximum theorem.

**Theorem 8.** *Suppose* $p < 1/2$. *There exist* $t_H^*$ *such that if* $t_H > t_H^*$ *then there exist* $q_{\min}, q_{\max} \in (t_L, \min\{q_l, t_H\})$ *such that* $\Delta(q) < 0$ *for* $q \in (q_{\min}, q_{\max})$ *and* $\Delta(q) \geq 0$ *for* $q \in (t_L, \min\{q_l, t_H\}) \setminus (q_{\min}, q_{\max})$.

*Proof.* The efficiency of group decision-making in equilibrium satisfies

$$E_{2n+1}[u^*] = \max_{(\sigma_1,\sigma_2,\sigma_3,\sigma_4)\in[0,1]\times[0,1]\times[0,1]\times[0,1]} E_{2n+1}[u](\sigma_1,\sigma_2,\sigma_3,\sigma_4)$$

The space $[0,1]\times[0,1]\times[0,1]\times[0,1]$ is compact and is independent of $t_H$. Then, we can apply the Berge's maximum theorem to conclude that $E_{2n+1}[u^*]$ is continuous in $t_H$. Similarly, the efficiency under the optimal choice behavior in single person decision-making $E_1[u^*]$ is continuous in $t_H$. Then, $\Delta(q)$ is continuous in $t_H$.

Under the supposition of $p < 1/2$, we claim that there exist $t_H^*$ such that for any $t_H \in (t_H^*, 1)$ there exist $q_{\min}(t_H), q_{\max}(t_H) \in (t_L, \min\{q_l, t_H\})$ such that $\Delta(q) < 0$ for $q \in (q_{\min}(t_H), q_{\max}(t_H))$. Suppose not. Then, there exists a sequence $\{t_H^n\}_{n=1}^{\infty}$ such that $\Delta(q, t_H^n) \geq 0$ for any $n$ and $q \in (t_L, \min\{q_l, t_H^n\})$. By Theorem 5, it holds that $\Delta(q, t_H^n) \geq 0$ for any $q \in [1/2, 1)$. Now we can take $\hat{q}$ such that $\Delta(\hat{q}, 1) < 0$ by Theorem 6. The continuity of $\Delta(\hat{q})$ in $t_H$ implies that $\Delta(\hat{q}, 1) = \lim_{n\to\infty} \Delta(\hat{q}, t_H^n) \geq 0$. This is a contradiction. □

Combining Theorem 5 and Theorem 8, we have the following.

**Proposition 3.** *Suppose $p < 1/2$. There exist $t_H^*$ such that if $t_H > t_H^*$ then there exist $q_{\min}, q_{\max} \in (t_L, \min\{q_l, t_H\})$ such that $\Delta(q) < 0$ for $q \in (q_{\min}, q_{\max})$ and $\Delta(q) \geq 0$ for $q \in [1/2, 1) \setminus (q_{\min}, q_{\max})$.*

Figure 6 illustrates Proposition 3. In Figure 6, we compare the efficiency of decision between the group decision-making by three members and the single person decision-making. we assume that $p = 0.1$, $t_L = 0.6$ and $t_H = 0.9$. Then, $q_l = 273/295 (\approx 0.9254)$, $q_h = 819/830 (\approx 0.9867)$ and $\bar{q} = 81/82 (\approx 0.9878)$. The optimal choice behavior is $(1,1,0,0)$ for $q \in [1/2, 0.6)$, $(1,0,0,0)$ for $q \in [0.6, 0.9)$ and $(0,0,0,0)$ for $[0.9, 1]$. The equilibrium voting behavior is $(1,1,0,0)$ for $q \in [1/2, 0.6)$, $(1, \sigma_q)$ for $q \in [0.6, 0.9254)$, $(1,0,0,0)$ for $(0.9254, 0.9867)$, $(\sigma_q, 0, 0, 0)$ for $q \in [0.9867, 0.9878)$ and $(0,0,0,0)$ for $q \in [0.9878, 1]$.

In addition to Proposition 3, we next examine the sufficient condition under which the superiority of the single person decision-making over the group decision-making by voting occurs monotonically with respect to the parameter $t_H$. We assume $p < 1/2$ as in Proposition 3. We consider $q^* \in \arg\min_{q\in(t_L, \min\{q_l, t_H\})} \Delta(q)$. The $q^*$ is unique since $\Delta(q)$ is convex in $q \in (t_L, \min\{q_l, t_H\})$ by Corollary 1 and Theorem 4. Then we show that $\Delta(q^*)$ is decreasing in $t_H$ for sufficiently large $t_L$.

**Theorem 9.** *There exist $t_L^*$ such that if $t_L > t_L^*$ then $\Delta(q^*)$ is decreasing in $t_H$.*

*Proof.* We show that

$$\lim_{t_L\to 1}\left[\max_{q\in(t_L,\min\{q_l,t_H\})} (1-q)\Pr(piv|\omega = A) + \max_{q\in(t_L,\min\{q_l,t_H\})} q \Pr(piv|\omega = B)\right] = 0.$$

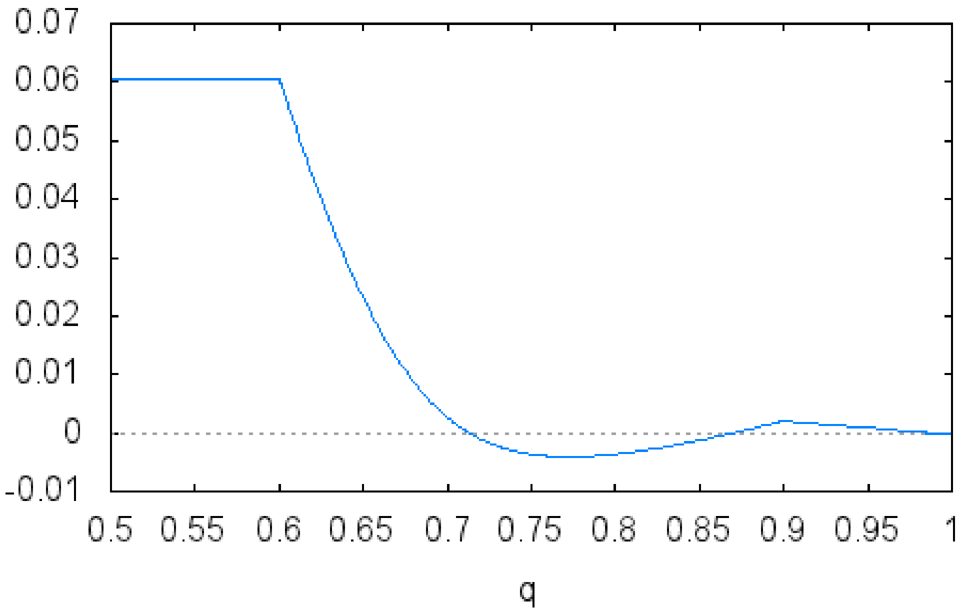


Figure 6: The difference of efficiency without the strong signal condition

Take any small $\epsilon > 0$. First, it holds that

$$\max_{q \in (t_L, \min\{q_l, t_H\})} (1-q) \Pr(piv|\omega = A) < \epsilon/2$$

for $t_L > t'_L(\epsilon) := 1 - \frac{\epsilon}{2}$, because

$$\begin{aligned} \max_{q \in (t_L, \min\{q_l, t_H\})} (1-q) \Pr(piv|\omega = A) &< \max_{q \in [t_L, \min\{q_l, t_H\})} (1-q) \times 1 \\ &= 1 - t_L \\ &< \epsilon/2. \end{aligned}$$

Second, it holds that

$$\max_{q \in (t_L, \min\{q_l, t_H\})} q \Pr(piv|\omega = B) < \epsilon/2$$

for $t_L > t_L''(\epsilon) := 1 - \left(\frac{\epsilon}{2\binom{2n}{n}}\right)^{1/n}$, because

$$\begin{aligned}
\max_{q\in(t_L,\min\{q_l,t_H\})} q\Pr(piv|\omega=B) &= \max_{q\in(t_L,\min\{q_l,t_H\})} q\times\binom{2n}{n}\gamma_B^n(1-\gamma_B)^n \\
&< \max_{q\in(t_L,\min\{q_l,t_H\})} \binom{2n}{n}(1-\gamma_B)^n \\
&= \max_{q\in(t_L,\min\{q_l,t_H\})} \binom{2n}{n}(p(1-t_H)+(1-p)(1-t_L)\sigma)^n \\
&< \binom{2n}{n}(p(1-t_L)+(1-p)(1-t_L))^n \\
&= \binom{2n}{n}(1-t_L)^n \\
&< \epsilon/2.
\end{aligned}$$

Let $t_L^*(\epsilon) = \max\{t_L'(\epsilon), t_L''(\epsilon)\}$, then

$$\max_{q\in(t_L,\min\{q_l,t_H\})}(1-q)\Pr(piv|\omega=A) + \max_{q\in(t_L,\min\{q_l,t_H\})} q\Pr(piv|\omega=B) < \epsilon$$

for $t_L > t_L^*(\epsilon)$. Thus, we have shown the desired fact. This fact implies

$$\lim_{t_L\to 1}(1-q^*)\Pr(piv|\omega=A) + q^*\Pr(piv|\omega=B) = 0.$$

Note that

$$\begin{aligned}
\frac{\partial}{\partial t_H}\Delta(q) &= \frac{\partial}{\partial t_H}[(1-q)(d_A-\rho_A)+q(d_B-\rho_B)] \\
&= (1-q)\left(\frac{\partial d_A}{\partial\gamma_A}\frac{\partial\gamma_A}{\partial t_H} - \frac{\partial\rho_A}{\partial t_H}\right) + q\left(\frac{\partial d_B}{\partial\gamma_B}\frac{\partial\gamma_B}{\partial t_H} - \frac{\partial\rho_B}{\partial t_H}\right) \\
&= (1-q)\left(\frac{(2n+1)!}{n!n!}[\gamma_A(1-\gamma_A)]^n p - p\right) + q\left(\frac{(2n+1)!}{n!n!}[\gamma_B(1-\gamma_B)]^n p - p\right) \\
&= p\left\{\frac{(2n+1)!}{n!n!}\{(1-q)[\gamma_A(1-\gamma_A)]^n + q[\gamma_B(1-\gamma_B)]^n\} - 1\right\} \\
&= p(2n+1)\left\{(1-q)\frac{2n!}{n!n!}[\gamma_A(1-\gamma_A)]^n + q\frac{2n!}{n!n!}[\gamma_B(1-\gamma_B)]^n - \frac{1}{2n+1}\right\} \\
&= p(2n+1)\left\{(1-q)\Pr(piv|\omega=A) + q\Pr(piv|\omega=B) - \frac{1}{2n+1}\right\}.
\end{aligned}$$

Therefore, $\lim_{t_L \to 1} \frac{\partial}{\partial t_H} \Delta(q^*) < 0$. Hence, there exist $t_L^*$ such that if $t_L > t_L^*$ then $\Delta(q^*)$ is decreasing in $t_H$.

□

Theorem 9 implies the following result.

**Corollary 2.** *Suppose $p < 1/2$ and $t_L > t_L^*$ where $t_L^*$ is the lower bound stated in Theorem 9. There exists $\tilde{t}_H$ such that*

- *if $t_H \leq \tilde{t}_H$, $\Delta(q) \geq 0$ for any $q \in [1/2, 1)$, and*
- *if $t_H > \tilde{t}_H$, there exist an interval of $q$ such that $\Delta(q) < 0$ as stated in Proposition 3.*

*Proof.* To express explicitly the fact that $\Delta(q)$ depends on $t_H$, we write it as $\Delta(q, t_H)$. Similarly, we write $q^*$ as $q^*(t_H)$.

Wit (1998) studied $\Delta(q, t)$ for the case of $t = t_H = t_L$. Wit (1998) showed that $\Delta(q, t) > 0$ when $q = t$. Then, we will show that there exists $t_H > t_L$ such that $\Delta(q^*(t_H), t_H) > 0$. First, it holds that $\lim_{t_H \to t_L} q^*(t_H) = t_L$, because $q^*(t_H)$ is in $[t_L, \min\{q_l, t_H\}]$ and $\min\{q_l, t_H\}$ goes to $t_L$ as $t_H$ goes to $t_L$. Moreover, $q^*(t_H)$ is continuous in $t_H$ by Berge's maximum theorem.*13 These facts imply $\lim_{t_H \to t_L} \Delta(q^*(t_H), t_H) = \Delta(t_L, t_L) > 0$. Therefore, there exists $t_H > t_L$ such that $\Delta(q^*(t_H), t_H) > 0$.

On the other hand, it holds that $\Delta(q^*(t_H), t_H) < 0$ for $t_H = 1$ by Proposition 1 and $p < 1/2$. Hence, by the continuity of $\Delta(q^*(t_H), t_H)$ in $t_H$, there exists $\tilde{t}_H$ such that $\Delta(q^*(\tilde{t}_H), \tilde{t}_H) = 0$. Moreover, by the monotonicity of $\Delta(q^*(t_H), t_H)$ under $t_L > t_L^*$ by Theorem 9, the $\tilde{t}_H$ is unique and if $t_H \leq \tilde{t}_H$, $\Delta(q) \geq 0$ and if $t_H > \tilde{t}_H$, there exist $q$ such that $\Delta(q) < 0$. Finally, the convexity of $\Delta(q)$ implies that the set $\{q | \Delta(q) < 0\}$ is an interval. □

Theorem 9 shows that $\Delta(q^*)$ is decreasing in $t_H$ for sufficiently large $t_L$. However, the monotonicity of $\Delta(q^*)$ in $t_H$ dose not hold generally. Figure 7 provides a counter-example. In Figure 7, we assume $p = 0.3$ and $t_L = 0.51$. The graphs of function 1 through function 4 show $\Delta(q)$ for $t_H = 1, 0.95, 0.6, 0.55$. In this example, $\Delta(q^*)$ is increasing over the range of $t_H = 0.55, 0.6$ and decreasing over the range of $t_H = 0.95, 1$.

In the example of Figure 7, we can not apply Corollary 2 directly to argue that if $t_H$ is low, the group decision-making is more efficient than single person decision-making for any prior probability $q$ and if $t_H$ is high, there exist a set of prior probabilities $q$ for which the single person decision-making is more efficient than group decision-making by voting. The next

*13Recall that $q^*$ is the unique minimizer of $\Delta(q, t_H)$ in the interval $[t_L, \min\{q_l, t_H\}]$. $\Delta(q, t_H)$ is continuous in $(q, t_H)$ and the correspondence from $t_H$ to $[t_L, \min\{q_l, t_H\}]$ is continuous. Therefore, by Berge's maximum theorem, the set of minimizers is an upper hemi-continuous correspondence. This means that the unique minimizer $q^*(t_H)$ is a continuous function.

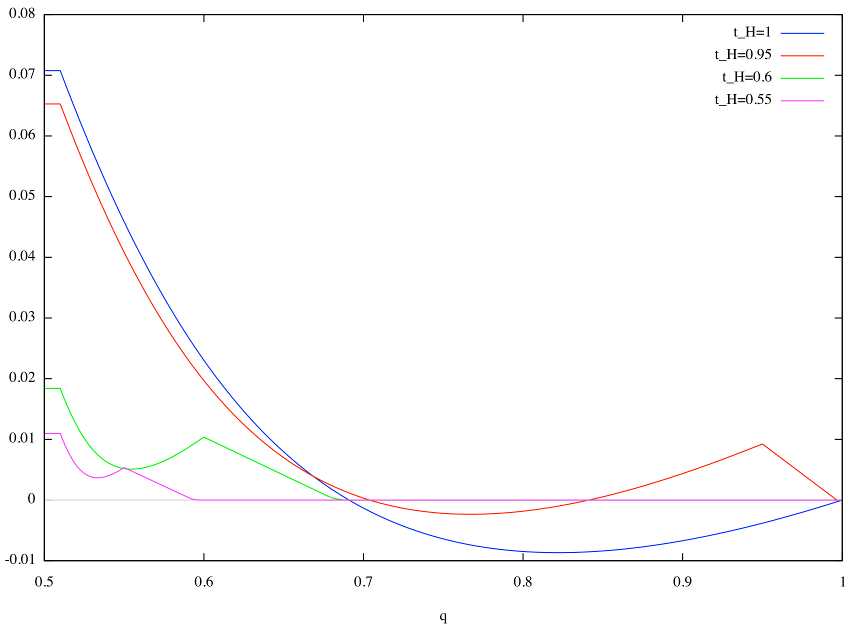


Figure 7: A counter-example of the monotonicity of $\Delta(q^*)$ in $t_H$.

example illustrates this issue for the entire space of $t_L$ and $t_H$. As long as this example is concerned, the monotonicity of the existence of superiority of single person decision-making with respect to $t_H$ holds for every value of $t_L$ from 0.51 to 1.

**Example** We provide an example of three-member committee with $p = 1/10$. In Table 1, S means that there exist $q$ such that $\Delta(q) < 0$ and G means that $\Delta(q) \geq 0$ for all $q \in [1/2, 1]$ for each $(t_H, t_L)$.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | | |
| 0.95 | | | | | | | | | | | S |
| 0.9 | | | | | | | | | | G | S |
| 0.85 | | | | | | | | | G | G | S |
| 0.8 | | | | | | | | G | G | G | S |
| 0.75 | | | | | | | G | G | G | G | S |
| 0.7 | | | | | | G | G | G | G | S | S |
| 0.65 | | | | | G | G | G | G | S | S | S |
| 0.6 | | | | G | G | G | G | S | S | S | S |
| 0.55 | | | G | G | S | S | S | S | S | S | S |
| 0.51 | | S | S | S | S | S | S | S | S | S | S |
| $t_L/t_H$ | 0.51 | 0.55 | 0.6 | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 | 1 |

Table 1: Example with $p = 1/10$

## 5.3 The Comparative Statics

Finally, we consider for what kind of decision problems the single person decision-making is more efficient than the group decision-making by voting. A decision problem is represented by its prior probability $q$. We focus on $q^*$ as "the center" of the set of decision problems for which the single person decision-making is more efficient than group decision-making by voting. We establish the following comparative statics of $q^*$ with respect to $t_H$.

**Theorem 10.** *$q^*$ is increasing in $t_H$*

*Proof.* It holds that $\Delta'(q^*) = 0$ since $q^* = \arg\min_{q \in (t_L, \min\{q_l, t_H\})} \Delta(q)$. For $q \in (t_L, \min\{q_l, t_H\})$, the voting behavior in equilibrium is $(1, \sigma_q, 0, 0)$ and the optimal choice behavior is $(1, 0, 0, 0)$.

First, recall that $\Delta(q)$ is convex in $q$ because $E_{2n+1}[u]$ is convex in $q$ and $E_1[u]$ is linear in $q$. Hence, $\frac{\partial}{\partial q}\Delta'(q) > 0$.

Second, recall that

$$\Delta'(q) = -[d_A - \rho_A] + [d_B - \rho_B].$$

by the envelop theorem. Then, we get

$$\begin{aligned}
\frac{\partial}{\partial t_H}\Delta'(q) &= \frac{\partial}{\partial t_H}[-(d_A-\rho_A)+(d_B-\rho_B)] \\
&= -\left(\frac{\partial d_A}{\partial \gamma_A}\frac{\partial \gamma_A}{\partial t_H}-\frac{\partial \rho_A}{\partial t_H}\right)+\left(\frac{\partial d_B}{\partial \gamma_B}\frac{\partial \gamma_B}{\partial t_H}-\frac{\partial \rho_B}{\partial t_H}\right) \\
&= -\left(\frac{(2n+1)!}{n!n!}[\gamma_A(1-\gamma_A)]^n p-p\right)+\left(\frac{(2n+1)!}{n!n!}[\gamma_B(1-\gamma_B)]^n p-p\right) \\
&= p\frac{(2n+1)!}{n!n!}\left(-[\gamma_A(1-\gamma_A)]^n+[\gamma_B(1-\gamma_B)]^n\right) \\
&= p(2n+1)\left(-\frac{2n!}{n!n!}[\gamma_A(1-\gamma_A)]^n+\frac{2n!}{n!n!}[\gamma_B(1-\gamma_B)]^n\right) \\
&= p(2n+1)\left(-\Pr(piv|\omega=A)+\Pr(piv|\omega=B)\right) \\
&< 0,
\end{aligned}$$

because $\frac{\Pr(piv|\omega=A)}{\Pr(piv|\omega=B)} > 1$ when the voting behavior is $(1,\sigma,0,0)$.

Therefore,

$$\frac{dq^*}{dt_H} = -\frac{\partial\Delta'(q^*)/\partial t_H}{\partial\Delta'(q^*)/\partial q^*} < 0.$$

□

# 6 Superiority of the Single Person Decision-making over all sub-committees

In section 5, we compared the efficiency between the single person decision-making and the group decision-making by voting. Under the group decision-making by voting, it is assumed that all the group members have the right to vote. However, the voting can be implemented not by letting all the group members vote but by letting some members in the group vote. A sub-committee decision-making by voting is a decision procedure in which $2m+1(1 \leq m \leq n)$ members are selected randomly from $2n+1$ committee members and the decision is made by their voting with the simple majority rule. In this section, we compare the efficiency between the single person decision-making and the decision-making by voting in general by considering all the sub-committee decision-making by voting. We define the difference of the efficiency of decision-making between $2m+1$ members subcommittee and the single person as

$$\Delta_1^{2m+1}(q) = E_{2m+1}[u] - E_1[u].$$

We show that there exists an interval of $q$ for which $\Delta_1^{2m+1}(q) < 0$ for $m = 1, 2, ..., n$. In words, we derive the condition under which the single person decision is superior to the decision-making by voting in general. Under this condition, when available decision-making procedures for the group are either the single person decision-making or the decision-making by voting with the simple majority rule in a suitably chosen committee, the single person decision-making is the best procedure.

Under the strong signal condition $t_H = 1$, Theorem 6 implies for a given $m = 1, 2, ..., n$ that if $p < 1/2$, there exists $\tilde{q}_m \in (t_L, 1)$ such that $\Delta_1^{2m+1}(q) < 0$ if and only if $q \in (\tilde{q}_m, 1)$, for any $m \geq 1$. This fact establishes the following result.

**Theorem 11.** *Suppose that $t_H = 1$ and $p < 1/2$. There exists $\hat{q} \in (t_L, 1)$ such that if $q \in (\hat{q}, 1)$ then $\Delta_1^{2m+1}(q) < 0$ for any $m = 1, 2, ..., n$.*

*Proof.* Define $\hat{q} \equiv \max_{m=1,2,...,n} \tilde{q}_m$. Then, $(\hat{q}, 1) = \cap_{m=1}^{n} (\tilde{q}_m, 1) \neq \emptyset$. Then, it holds for any $m = 1, 2, .., n$ that $\Delta_1^{2m+1}(q) < 0$ for $q \in (\hat{q}, 1)$. □

When $t_H < 1$, Theorem 8 implies for a given $m = 1, 2, ..., n$ that if $p < 1/2$, there exists $t^*_{H,m}$ such that if $t_H > t^*_{H,m}$, then there exists an interval $I_m(t_H) = (q_{\min,m}(t_H), q_{\max,m}(t_H))$ such that $\Delta_1^{2m+1}(q) < 0$ if and only if $q \in I_m(t_H)$. By Theorem 8 and Theorem 11, we establish Theorem 12 below. Theorem 12 states that when $t_H$ is sufficiently high, there exists an interval of $q$ for which the single person decision is superior to the decision-making by voting in general.

**Theorem 12.** *Suppose that $p < 1/2$. There exists $t^{**}_H$ such that if $t_H > t^{**}_H$, then there exists an interval $I(t_H)$ such that if $q \in I(t_H)$, then $\Delta_1^{2m+1}(q) < 0$ for any $m = 1, 2, ..., n$.*

*Proof.* We prove by contradiction. Suppose there does not exist $t^{**}_H$ stated in Theorem 12. Then, there exists a sequence $\{t^k_H\}$ such that [1] $t^k_H \to 1$, and [2] $I(t^k_H) \equiv \cap_{m=1}^{n} I_m(t^k_H) = \emptyset$ for any $k$ where $I_m(t_H)$ is an interval stated before Theorem 12. Fix $q \in (\hat{q}, 1)$ where $\hat{q}$ is the threshold stated in Theorem 11. Then, $q \notin I(t^k_H)$ for any $k$ since $I(t^k_H) = \emptyset$ for all $k$. This implies that there exist a sequence $\{m_k\}$ such that $\Delta_1^{2m_k+1}(q, t^k_H) \geq 0$ for all $k$. Here, we can take a subsequence $\{m_{k_l}\}$ such that $m_{k_l} = m'$ for all $l$ and for some $m' \in \{1, 2, ..., n\}$, because the sequence $\{m_k\}$ is taken in the finite set $\{1, 2, ..., n\}$. By the continuity of $\Delta_1^{2m'+1}(q, t_H)$ in $t_H$, it holds that

$$\begin{aligned} 0 &\leq \lim_{l\to\infty} \Delta_1^{2m'+1}(q, t^{k_l}_H) \\ &= \Delta_1^{2m'+1}(q, \lim_{l\to\infty} t^{k_l}_H) \\ &= \Delta_1^{2m'+1}(q, 1). \end{aligned}$$

This is a contradiction to the fact that $\Delta_1^{2m+1}(q, 1) < 0$ for any $m = 1, 2, ..., n$ when $q \in (\hat{q}, 1)$. □

# 7 Concluding Remarks

In order to understand the logic of the possible superiority of single person decision-making, we assumed that there are two types of the degree of information precision; the higher type and the lower type. However, it is easy to extend the model to a general model in which there are $l(\geq 2)$ types of the degree of information precision. In the general model, the superiority of single person decision-making also holds under the conditions corresponding to those presented in this paper. The reason for the superiority of single person decision-making is also the same.

# References


[1] David Austen-Smith and Jeffrey S Banks. Information aggregation, rationality, and the Condorcet jury theorem. *American Political Science Review*, 90(01):34–45, 1996.

[2] Ruth Ben-Yashar and Jacob Paroush. A nonasymptotic Condorcet jury theorem. *Social Choice and Welfare*, 17(2):189–199, 2000.

[3] John Duggan and César Martinelli. A bayesian model of voting in juries. *Games and Economic Behavior*, 37(2):259–294, 2001.

[4] Timothy Feddersen and Wolfgang Pesendorfer. Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting. *American Political science review*, 92(01):23–35, 1998.

[5] Timothy J Feddersen and Wolfgang Pesendorfer. The swing voter's curse. *The American economic review*, pages 408–424, 1996.

[6] Kerstin Gerling, Hans Peter Grüner, Alexandra Kiel, and Elisabeth Schulte. Information acquisition and decision making in committees: A survey. *European Journal of Political Economy*, 21(3):563–597, 2005.

[7] JMM Goertz. Inefficient committees: Small elections with three alternatives. *Social Choice and Welfare*, 43(2):357–375, 2014.

[8] Li Hao and Wing Suen. Decision-making in committees. *Canadian Journal of Economics*, 42(2):359–392, 2009.

[9] Andrew McLennan. Consequences of the Condorcet jury theorem for beneficial information aggregation by rational agents. *American Political Science Review*, 92(02):413–418, 1998.

[10] Joseph C McMurray. Aggregating information by voting: The wisdom of the experts versus the wisdom of the masses. *The Review of Economic Studies*, 80(1):277–312, 2013.

[11] Adam Meirowitz. Informative voting and Condorcet jury theorems with a continuum of types. *Social Choice and Welfare*, 19(1):219–236, 2002.

[12] Kaushik Mukhopadhaya. Jury size and the free rider problem. *Journal of Law, Economics, and Organization*, 19(1):24–44, 2003.

[13] Jörgen Wit. Rational choice and the Condorcet jury theorem. *Games and Economic Behavior*, 22(2):364–376, 1998.