Contribute once! Full efficiency in a dynamic contribution game*

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In simple dynamic contribution games with a finite discrete time, it is often assumed that repetitive contributions are allowed. Then, while efficient outcomes are achievable at a subgame perfect equilibrium (SPE), an inefficient outcome is also achievable at another SPE. This study considers a case in which any continuous contribution is allowed, but on the condition that each player can contribute only once. The study then shows that all SPEs achieve efficiency if and only if the number of periods is greater than or equal to the number of players. More importantly, each SPE is essentially unique in the sense that the provision is completed in the first period.

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1. Introduction

Typically, when raising a fund to finance public projects, the fundraiser collects voluntary donations from potential donors. This situation of voluntary donation is usually described as a dynamic contribution game (Andreoni, 1998; Name-Correa and Yildirim, 2013). A dynamic contribution game is a multiperiod game with the following features: (a) in each period, each donor can donate any amount and (b) once the total donations hit the predetermined threshold, a public good is provided. Designing a voluntary contribution scheme to achieve efficient outcomes is an important aspect of the fundraising design.

Prior studies reveal that dynamic contribution games generally possess many equilibria, including inefficient ones. Marx and Matthews (2000) show that while an efficient outcome is achieved at a subgame perfect equilibrium (SPE), an inefficient outcome also survives. In other words, no contribution is made on the equilibrium path in a SPE.

This study aims to design a voluntary contribution scheme that has only efficient equilibria. One feature in standard dynamic contribution games is that players can contribute as many times as they want. Although seemingly harmless, this feature may cause an inefficient outcome. Kamdar et al. (2015) may be experimental evidence of this possibility. They propose an experimental treatment in a charity situation, called *once and done*. In this treatment, each potential donor receives a mail that states, “Make one gift now and we will never ask for another donation again.” Kamdar et al. (2015) find that the once-and-done treatment doubles the amount of the initial donation. This experimental fact suggests that imposing a restriction that each player can contribute only once, improves the efficiency of the outcomes of fundraising campaigns.

Our model incorporates this idea into a standard dynamic contribution game: we study a discrete time multiperiod game with perfect information, in which each player can make any amount of contribution at any period, but *only once*. Once the total amount of contribution hits the predetermined threshold, a public good is provided. Each player has a valuation for the provision. The cost of contribution equals the amount of contribution.

Under the “contribute-once” restriction, the study shows that an efficient outcome is achieved in any SPE if and only if the number of periods is greater than or equal to the number of players. More importantly, SPEs are essentially unique in the sense that any contribution is made in the first period. This maximizes the social welfare of the players.

We provide an intuition for the main logic behind the efficiency and no-delay result of this study. Consider a two-donor case. Completing the provision is efficient if and only if the sum of these two donors’ valuations exceeds the required amount of contribution. We assume that completing the provision is efficient. To avoid trivial results, we also assume that the required amount exceeds the valuation of each single donor. In a static game, there exists an inefficient equilibrium; no one makes any positive contribution. As the other makes no contribution, to complete the provision, one donor needs to contribute an amount that exceeds his valuation for the provision. Thus, each donor has no incentive to contribute. Even in a dynamic case, without the contribute-once restriction, Marx and Matthews (2000) show that such a no-contribution outcome occurs in a SPE. In their proposing strategy, after any unilateral deviation, the deviating donor needs to complete the provision alone, while the others make no contribution afterward. This implies that the full cost of financing the public good is borne only by the deviating player. Thus, any deviation is unprofitable.

However, under the "contribute-once" scheme, there is no such SPE. Consider two donors, A and B, and two contribution periods, and suppose there is a SPE with zero contributions. Then, by definition, no one contributes in the first period. In this case, at least one donor finds a profitable deviation. Consider the followings. Donor A makes a contribution in the first period such that the remaining amount required for completing the provision is less than donor B's valuation. Since the sum of the valuation of donor A and B exceeds the total amount of required contribution, this deviation only requires a cost that is less than donor A's valuation. Thus, if donor B completes the provision in the second period, this deviation is profitable. After this deviation, in the second period, because of the "contribute-once" restriction, donor A cannot make any positive contribution. Thus, only donor B is able to complete the provision. Then, as the remaining amount is less than the valuation, completing the provision is the unique optimal choice for donor B. The provision is then completed, which profits donor A. Hence, no contribution is never a SPE outcome. This logic can be extended to n-donor, n-period cases with an induction.

Delayed completion cases also allow for a profitable deviation. Suppose that both donors make positive contributions in the second period.[1] As the public good is provided, each player obtains a positive surplus. In this case, the following deviation is profitable. In the first period, donor A contributes an amount that is slightly less than the original one. By doing so, donor B still has an incentive to complete the provision, and then, donor A neatly reduces the burden. As a result, in any SPE, the completion is done in the first period.

Our result presents an implication for the design of charity and fundraising programs. In addition to allowing multiple periods for donations and any amount of donations, an individual is allowed to donate only once. This restriction causes every SPE outcome to be efficient. One may

[1] If only one donor contributes in the second period, he can hasten completion by contributing the same amount in the first stage, which improves his utility.

question the feasibility of the restriction. Indeed, for a fundraiser, it is hard to decline a second or third donation from one individual. One possibility of overcoming this difficulty is to use a fundraising agency that provides a fundraising service adopting the "contribute-once" scheme. Accepting a second donation has a reputational effect on the agency; fundraisers and potential donors believe that this "contribute-once" scheme is titular. This may result in an inefficient outcome in another donation campaign. Consequently, the agency loses credit with fundraisers and loses future profits. This incentivizes the agency to refuse a second donation from the same individual. This argument suggests that our "contribute-once" scheme is implementable in practice.

2. Literature review

As shown by Matthews (2013), dynamic contribution games generally possess many equilibria, including an inefficient one.[2] Several studies show the uniqueness and efficiency of the equilibria of contribution games allowing repetitive contributions. Related to this point, Bagnoli and Lipman (1989) construct a mechanism that implements the core allocations in a type of trembling-hand perfect equilibrium.

Admati and Perry (1991) consider the case of alternative contributions and show the uniqueness and inefficiency of the SPE. Compte and Jehiel (2003) consider a variant of the model presented by Admati and Perry (1991) and show the uniqueness of the SPE, although it is efficient. Similar to our results, Marx and Matthews (2000) show that, in the setting of Admati and Perry (1991), by adding a finite deadline, the public good is provided in all SPE outcomes. In contrast to our results, however, they do not show the relation between the length of the periods and number of players, since there are only two players in Admati and Perry's (1991) model. Furthermore, in contrast to ours, in the variants of Admati and Perry's (1991) model, a sufficiently high discount factor is needed to generate their efficiency results.

Another approach to achieving uniqueness and efficiency is admitting stochastic noisy progress and gradual adjustment in continuous time in the accumulated contribution (Georgiadis, 2015; Iijima and Kasahara, 2016). In such settings, the contribution made by players is indistinguishable from that produced by the stochastic noise. In addition, since the decision is made in a continuous time, the actions of others have no effect on the current amount of the accumulated contribution. This destroys the no-contribution equilibrium. Indeed, in a general form of dynamic contribution games with continuous time, Iijima and Kasahara (2016) show

[2] Matthews (2013) allows repetitive contributions.

the uniqueness and efficiency of the Nash equilibrium.

A difference between those studies and mine is that, in dynamic contribution games with stochastic noise, it is necessary to gradually adjust the contribution amount. In contrast, we show that, in each SPE, the provision is completed immediately, and, thus, it admits no gradualism. This is the novelty of this study.

Considering contribution games with gradual adjustment in continuous time, Cvitanić and Georgiadis (2016) characterize a mechanism achieving efficient outcomes in the Markov perfect equilibrium (MPE). Although the focus of many studies in contribution games is a non-durable public good, uniqueness and efficiency are established for the provision of durable public goods. Battaglini, Nunnari, and Palfrey (2014) show that, following a symmetric MPE, if the depreciation rate is sufficiently small, the equilibrium steady state converges to a unique and efficient point.

Our efficiency result depends on the discrete public good structure. In studies of continuous public goods, such as those by Varian (1994) and Romano and Yildirim (2001, 2005), the contribute-once restriction leads to a decline in total contribution. Under the continuous public good structure, the contribute-once restriction also enables a first-mover advantage, which increases free riding on the second-mover's contribution. As the public good is continuous, the first-mover's contribution becomes parsimonious. However, if the public good is discrete, in the case of not completing the provision, the first-mover's contribution is wasted. As a result, no equilibrium with such parsimonious contributions exists. On the contrary, since the first-mover advantage remains, each player's contribution is hastened.

The contribute-once assumption restricts contributors' choices. As another way to restrict their choices, Barbieri and Malueg (2014) consider a restriction under which only binary contributions are allowed. In their static and incomplete information setting, they show that this restriction increases the probability of fundraising success.

Lastly, our result does not depend on the amount of the discount factor. This feature is uncommon in dynamic games.

3. The model

The model consists of n players. Let $N = \{1, \ldots, n\}$ denote the set of players. Time is discrete and bounded by the deadline $\bar{T} \in \mathbb{N} \cup \{\infty\}$ and $\mathcal{T} := \{1, \ldots, \bar{T}\}$. Player $i \in N$ has valuation $V_i > 0$ of the provision of a discrete public good. To provide the discrete public good, the players need to contribute $X^* > 0$ in total. The contribution cost equals the contribution amount. For player $i \in N$, let $z_i(t) \geqslant 0$ be the contribution amount of player i in period $t \in \mathcal{T}$. Let $z = (z_i(t))_{i \in N, t \in \mathcal{T}}$ denote the contribution profile. Also let $T(z) := \inf\{t \in \mathcal{T} \cup \{\infty\} \colon X(t) \geqslant X^*\}$ be the completion period. For brevity, let $T(z) = \infty$ if the provision is not completed (i.e., $\{t \in \mathcal{T} \colon X(t) \geqslant X^*\} = \varnothing$). The cost of the committed contribution is borne in the completion period or upon the deadline.[3] If the provision is not completed by the deadline, the proportion $\alpha \in [0, 1]$ of the contributions is refunded. Finally, let $\delta \in (0, 1)$ denote the discount factor.

Then, for each contribution profile z and each $i \in N$, player i's utility is given by

$$u_i(z) := \delta^{T(z)-1} V_i - \left((1 - \alpha) \delta^{\min\{T(z), \bar{T}\}-1} + \alpha \delta^{T(z)-1} \right) \sum_{t=1}^{\min\{T(z), \bar{T}\}} z_i(t).$$

Let us explain this. $T(z) = 1$ implies the immediate provision of the public good. Then, $u_i(z) = V_i - z_i(1)$. Similarly, if the provision is completed in period $\tilde{t} \leqslant \bar{T}$, $u_i(z) = \delta^{\tilde{t}-1}(V_i - \sum_{t=1}^{\tilde{t}} z_i(t))$. Recall that $T(z) = \infty$ implies that the provision is not completed by the deadline. Then, the players only pay their committed contribution upon the deadline, whereas the proportion α of their contribution is refunded. Now, player i's utility is $-(1 - \alpha)\delta^{\bar{T}-1} \sum_{t=1}^{\bar{T}} z_i(t)$. For example, $\alpha = 0$ is the fully nonrefundable case. On the contrary, if $\alpha = 1$, each player's committed contribution is fully refundable. If the provision is not completed, each player pays nothing. In addition, $\alpha \in (0, 1)$ is possible. For example, although the committed contribution is refundable, it may require a commission fee.

The above setting is a variant of one presented by Marx and Matthews (2000).[4] The following assumption is new and crucial to our contribution.

Assumption 1 (Contribute once)**.** For each $i \in N$ and $t \in \mathcal{T}$, if $z_i(t) > 0$, for each $t' \in \mathcal{T} \setminus \{t\}$, $z_i(t') = 0$.

This assumption implies that each player can contribute only once and no further contribution is allowed. This is a realistic restriction. For example, consider a fundraising campaign. A direct mail calling for donations typically encloses one transfer form for each donation. Hence, donations are effectively performed only once. However, fundraisers have some commitment issues about this restriction. They may be tempted to solicit additional donations, which benefits the fundraisers ex post, and such restriction becomes titular. However, this problem can be solved in a repeated game situation. See Section 5.3 for details.

[3] Although our framework may seem nonstandard, this specification is possible in charity and fundraising. For example, while each donation is committed on the decision date, withdrawal is made in the completion period $T(z)$ or upon the deadline $\bar{T}$. See also Appendix B.3 for a discussion on this assumption.

[4] In the work by Marx and Matthews (2000), $u_i(z) = \delta^{T(z)-1} V_i - \sum_{t=1}^{T(z)} \delta^{t-1} z_i(t)$.

We further assume that, in each period t, each player observes the previous amounts of committed contributions $Z(t)$ and the set of active players.[5] Let $N(t)$ be the set of active players in period t. Under Assumption 1, for each $t \in \mathcal{T}$, $N(t) = \{i \in N : \sum_{\tau<t} z_i(\tau) = 0\}$. Without Assumption 1, $N(t) = N$ for each $t \in \mathcal{T}$.

This study focuses on the *pure-strategy SPE* in which each player i never takes a strategy calling for $z_i(t) \geqslant V_i$.[6] If a player takes such a strategy on the equilibrium path, the payoff is at most zero, and, therefore, such a strategy cannot be better than not contributing in any subgame.[7] Thus, this refinement is natural. To keep the notation simple, this solution concept is abbreviated to SPE throughout this paper.

4. Characterization of equilibria

Under Assumption 1 and some conditions, each SPE achieves efficiency.

Theorem 1. *Suppose Assumption 1 holds and $X^* < \sum_{i \in N} V_i$.*

(a) There exists a SPE such that $\sum_{i=1}^{n} z_i(1) = X^*$ and $z_i(1) \in (0, V_i)$ for each $i \in N$ (i.e., the provision is completed in period 1).

(b) If $\alpha < 1$, $\bar{T} < \infty$, and $\bar{T} \geqslant n$, in each SPE, the provision is completed in period 1.

(c) Let $V^{(k)}$ be the k-highest valuation for each $k = 1, \dots, n$. Suppose that $X^* > \sum_{k=1}^{\bar{T}} V^{(k)}$. Then, there is a SPE in which no contribution is made on the equilibrium path.

Proof of Theorem 1. See Appendix A.1. □

This theorem states that only the immediate provision is the SPE outcome. Although the existence of an efficient SPE (Theorem 1 (a)) is not new,[8] the no-delay result (Theorem 1 (b)) is new. We briefly provide an intuition for this result.

First, we see the reason why the provision is necessarily completed. At first, suppose that, in the last period, there is only one active player. Apparently, he completes the provision if and only if the remaining amount is less than his valuation. Second, suppose that, in period $\bar{T} - 1$,

[5] The observability of the set of active players is necessary. See the discussion in Appendix B.5.

[6] In Appendix B.2, a case without this restriction is discussed.

[7] Under Assumption 1, this equilibrium refinement differs from eliminating weakly dominated strategies. To clarify, consider a strategy such that $z_i(t) = V_i$ only when $N(t) = \{i\}$ and $R(t) = V_i$; otherwise, $z_i(t) = 0$. This strategy is not weakly dominated by that of not contributing in any subgame. These strategies are indifferent for player i.

[8] In the static case, the existence of an efficient equilibrium is well known. See Palfrey and Rosenthal (1984) for a binary contribution and Nitzan and Romano (1990); Andreoni (1998) for a continuous contribution. Without Assumption 1, Fact 1 in the next subsection guarantees the existence of such a SPE.

there are two active players. Suppose also that there is a non-completing equilibrium. Note that a player can deviate to contribute in period $\bar{T} - 1$ so that the remaining amount is slightly less than the other player's deviation. Then, the deviating player can make no more contributions in the next period, due to the contribute-once restriction. Now as we have seen above, in period $\bar{T}$, completing the provision is unique optimal behavior for the non-deviating player, and, thus, the deviating player gains a positive utility, which is better than the non-completing outcome. As a result, non-completion is never a SPE outcome. With an induction, we can extend this intuition for general n-player cases when $n \leqslant \bar{T}$.

The no-delay result is followed by the above result. To see this, suppose, by contradiction, that there is a SPE in which the provision is completed in period $T \geqslant 2$. If there is only one player contributing in period T, he can profitably deviate by contributing the same amount in period $T - 1$. If there are two or more players contributing in period T, one of them can profitably deviate by contributing a slightly smaller amount in period $T - 1$. By the contribute-once assumption, it forces the other players to contribute more in period T. As the provision is necessarily completed, this implies that the provision is completed in period $T = 1$.

For this result, the assumption $\bar{T} \geqslant n$ plays an important role. This assumption is necessary for the efficiency of SPE outcomes. If $n > \bar{T}$, $\sum_{i \in N} V_i > X^* > \sum_{k=1}^{\bar{T}} V^{(k)}$ might be possible. Then, in a SPE, no contribution is made on the equilibrium path (Theorem 1 (c)). This outcome is inefficient. Thus, each SPE outcome is efficient for any situation if and only if $\bar{T} \geqslant n$. This is a complete characterization to guarantee the efficiency of all SPE outcomes. The condition $\bar{T} \geqslant n$ is not restrictive. For example, it is satisfied if time is continuous.

There is another interesting feature under Assumption 1.

Proposition 1. *Suppose that* $V_i > (1-\delta)V_j$ *for each* $i, j \in N$, $X^* \in ((1-\delta)\max_{j \in N} \sum_{i \in N \setminus \{j\}} V_i, \sum_{i \in N} V_i)$,[9] $\alpha < 1$, $n \leqslant \bar{T} < \infty$, *and Assumption 1 holds. Then, in each SPE,* $z_i(1) > 0$ *for each* $i \in N$.

Proof. See Appendix A.2 □

As $\delta \to 1$, for any $X^* \in (0, \sum_{i \in N} V_i)$, each SPE forces all players to make a positive contribution in the initial period. This result slightly refines the set of contribution profiles in the SPE by using Assumption 1. Given this, each SPE excludes extremely unequal outcomes such that the provision cost is borne by only a proper subset of players but no burden is imposed on the others. Without Assumption 1, such an unequal outcome can be realized as a SPE (see

[9] The assumption that $X^* > (1 - \delta)\max_{j \in N} \sum_{i \in N \setminus \{j\}} V_i$ is necessary. To see this, consider that $N = \{a, b, c\}$, $V_a = V_b = V_c = V$ and $X^* < 2(1 - \delta)V$. Then, the contribution profile, such that players a and b contribute $X^*/2 < (1 - \delta)V$, and player c makes no contribution in the first period, is a SPE outcome. Under this contribution profile, players a and b's gain is more than δV. Any deviation by player a or b implies a delayed provision. Thus, their gain is at most δV, and, thus, the deviation is unprofitable.

Fact 1 in Appendix B.1). In such a SPE, noncontributing players free ride on contributing players. This observation suggests that Assumption 1 also alleviates the free-rider incentive.

5. Discussion

5.1. Tightness of the assumptions

Our main result, Theorem 1(b), depends on the following six main assumptions. First, each player can only contribute once. Second, $z_i(t) < V_i$ for each $i \in N$ and $t \in \mathcal{T}$. Third, the payment is made on completion period. Fourth, $\alpha < 1$. Fifth, that $\bar{T} < \infty$. And, finally, the identity of each player, together with the amount of the contribution, is common knowledge. In the following, we explain why these assumptions are essential to our results.

The contribute-once assumption is the most important one among these assumptions. Without this, we admit multiple equilibria including non-completing and delayed completion equilibria (a detailed discussion is given in Appendix B.1). The intuition behind the difference is the following. When the players can contribute as many times as they want, a punishment strategy becomes available, which enables a vast number of contribution profiles, including the non-completion equilibrium. The punishment strategy is the following. If a player deviates from a predetermined contribution profile, the others never contribute after his deviation. Then, the player needs to complete the provision only by himself, which is unprofitable. Notice that, to implement the non-completion outcome, a deviating player needs to be able to contribute twice; once at the time of the deviation, and once at the time of the punishment. The contribute-once assumption disables the punishment strategy, and it also disables the non-completion equilibrium.

The assumption that each player is restricted to play strategies calling for $z_i(t) \in [0, V_i)$ for each $i \in N$ and $t \in \mathcal{T}$ is necessary for the no-delay result. Without this, as it allows indifference between completing and not completing the provision, our game admits an equilibrium in which the provision is completed in the second or later periods, but the delay is bounded (see Appendix B.2 for an example).

The assumption about the withdrawal date also plays an important role in the no-delay result. In a canonical setting, the cost of contribution is borne at the period of contribution. Nevertheless, if $\delta \approx 1$, the time at which the cost of contribution is borne has little effect on the utility of the player. Then, our results remain valid. However, in such a setup, if δ is small, as the earlier contribution is costly, a SPE with delayed completion appears (see Appendix B.3 for an example).

Although violations of the above two assumptions admit a delay in completion of the provision, as long as providing the public good is efficient, the delay is bounded. Indeed, as the length of the periods converges to zero, the delay in the completion is virtually zero (see Appendix C for the formal statement).

The above-mentioned two assumptions play a role in the no-delay result; however, they are not critical for the efficiency result (i.e., the provision is completed in each SPE). On the other hand, the assumption that $\alpha < 1$ plays a role in our efficiency result. If $\alpha = 1$, as any contribution is fully refunded on failure of completion, the contribution profile that fails to reach the required amount is indifferent to no contribution. This indifference admits a SPE in which each player makes a positive but small contribution such that the sum fails to reach the requirement. Then, the provision is not completed on the equilibrium path (see Appendix B.4).

In our model, as the cost of contribution is borne in the completion period or upon the deadline $\bar{T}$, $\bar{T} = \infty$ creates the same situation as $\alpha = 1$. That is, when the provision fails, the cost of contribution is never borne. Hence, $\bar{T} < \infty$ is also necessary for our efficiency result.[10] Remarkably, as long as $n \leqslant \bar{T} < \infty$, the size of $\bar{T}$ is irrelevant to the date of the provision; it is completed in period 1.

The observability of the set of active players $N(t)$ is also essential to the efficiency result. Without this assumption, there is an equilibrium in which no contribution is made on the equilibrium path (an example is presented in Appendix B.5). That is, any deviation from the no-contribution profile is unprofitable. This is because, when the contribution profile is unobservable, for each unilateral deviation to contribute a positive amount, the active players can mistakenly believe that too many players contribute for the deviated amount.[11] Then, the players also believe that the number of players becomes too few to complete the provision at that time. Then, the provision is never completed, and, thus, the deviation is a waste of the money. As a result, the no-contribution profile becomes an equilibrium. This observation suggests that, to implement efficient outcomes, the fundraiser needs to announce not only the contribution amount, but also the set of contributors to date.

5.2. Extensions

Although this study focuses on a specific contribution game, some of our results can be extended to more general cases. First, while we consider a discrete public good, Marx and Matthews

[10] In the model where any cost of contribution is borne at the time of contribution, the $\bar{T} = \infty$ case is different from $\alpha = 1$. In this case, efficiency is also guaranteed when the length of the periods is sufficiently short. See Appendix C.

[11] It is possible because off-path belief is arbitrary in a perfect Bayesian equilibrium.

(2000) adopt a more general framework. In their model, the valuation of a public good takes a form such that

$$f_i(X) = \begin{cases} \lambda_i X & \text{for} \quad X < X^*, \\ V_i & \text{for} \quad X \geqslant X^*, \end{cases} \tag{1}$$

where X is the amount of the total contribution and $\lambda_i \leqslant V_i/X^* < 1$. That is, although the required amount of contribution is not collected by the deadline, they receive a partial benefit from their contribution. When $\lambda_i = 0$ for each $i \in N$, the model is reduced to ours. Under this framework, our results still hold for sufficiently small $\lambda_i \geqslant 0$. See Appendix D.1 for the formal statement and the proof.

Second, the main result also depends on the assumption of symmetric information. To consider an extension in this direction, we provide a two-type model: for each player $i \in N$, valuation V_i is randomly drawn from $\{V_L, V_H\}$ and the private value of player i. We focus on the type-symmetric perfect Bayesian Nash equilibrium (PBNE). In a two-player setting with the assumption that $2V_L > X^*$, under Assumption 1, we can show that the provision is completed in the first period with probability 1. The formal analysis is presented in Appendix D.2.

5.3. Practical implication

Our main result, Theorem 1 presents a practical implication for the design of fundraising problems. In addition to allowing multiple periods for donations and of any amount, an individual is allowed to donate only once. Then, in any SPE, the provision is completed immediately and efficiently.

On the other hand, the implementation of the contribute-once scheme seems difficult in practice. A charitable fundraiser would have no incentive to decline a second or further contribution from the same individual. He may also have an incentive to solicit further contributions ex post. Therefore, some commitment device is necessary to realize the "contribute-once scheme."

One solution to this commitment problem is to use a fundraising agency that conducts fundraising campaigns on behalf of the fundraisers. The agency adopts the "contribute-once" scheme, which guarantees the efficiency of the fundraising campaign, as demonstrated. Although each individual fundraiser may conduct a campaign only once or twice, the agency operates over the long-term: it has many opportunities to conduct fundraising campaigns. This creates a repeated game situation. The agency then implements the contribute-once scheme to maintain its reputation.[12]

[12] The following argument is familiar in the literature on repeated games (see Mailath and Samuelson, 2006).

Let us examine this scenario in detail. The agency has an incentive to implement the "contribute-once" scheme, that is, to decline additional contributions from the same individual in each campaign. Although it may benefit the campaign fundraiser if the agency receives or solicits additional contributions, it also damages the agency's reputation. Future fundraisers and potential donors might believe that the agency would renege on the "contribute-once" scheme. This may result in an inefficient outcome for another campaign. Consequently, future fundraisers would never delegate their campaigns to this agency, which would hurts the agency's future profits.[13] Therefore, declining (and not soliciting) any additional contributions is optimal. Because of this possibility, the "contribute-once" scheme is implementable.

6. Conclusion

In this study, we focus on dynamic contribution games for a discrete public good with a finite discrete time, which generally possess many equilibria. As shown in Marx and Matthews (2000), if any repetitive contribution is allowed, an inefficient SPE survives. We show that, by banning repetitive contributions, only the efficient outcome survives. Hence, from the perspective of a practical application, our result suggests that inefficient outcomes can be avoided by allowing only one donation, albeit of an arbitrary amount.

One limitation of this study is that it assumes complete and perfect information. Although a parsimonious extension in this direction is considered, the analyses are insufficient for practice. Indeed, in the incomplete information case that we consider, we make a too strong assumption as it implies that, in any valuation profile, the completion is efficient. We leave the challenge of coping with these limitations to future research.

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[13] This is a type of grim-trigger strategy.

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A. Proofs in Section 4

A.1. Proof of Theorem 1

Part A: Proof of (a) Consider a contribution profile such that $\sum_{i=1}^{n} z_i(1) = X^*$ and $z_i(1) \in (0, V_i)$ for each $i \in N$. Then, under the contribution profile, each player gains a positive payoff.

To show that this is a SPE contribution profile, consider a deviation by player $j \in N$. A deviation such that $z'_j(1) > z_j(1)$ is unprofitable, as it only increases cost. Consider a deviation such that $z'_j(1) < z_j(1) < V_i$. Then, the provision is not completed in period 1.

Recall that $N(t) = \{i \in N\colon \sum_{\tau<t} z_i(\tau) = 0\}$ is the set of active players in period t. From the definition of the contribution profile, after j's deviation, in period 2, $N(2) \setminus \{j\} = \emptyset$. Therefore, no other player can make a positive contribution. To complete the provision, it is necessary that $\sum_{t\geqslant 2} z'_j(t) \geqslant z_j(1)$. However, under such a contribution profile, player j's profit is no greater than that under the original contribution profile. Therefore, player j's deviation is unprofitable. □

Part B: Proof of (b) The proof is an induction. Recall that $N(t) = \{i \in N\colon \sum_{\tau<t} z_i(\tau) = 0\}$ is the set of active players, and let $n(t) = |N(t)|$. Recall also that $T(z)$ is the completion period. For convenience, let $X(t) \coloneqq \sum_{\tau=1}^{t} \sum_{i=1}^{n} z_i(\tau)$, $R(t) \coloneqq X^* - X(t-1)$ for each $t > 1$ and $R(1) \coloneqq X^*$. $R(t)$ is the remaining amount of contribution needed to complete the provision at period t.

Step 1. Consider a subgame in period t such that $n(t) = 1$, $V_i > R(t)$ ($\{i\} = N(t)$), and $t \leqslant \bar{T}$. Then, in each SPE of this subgame, $T(z) = t$.

Proof of Step 1. As $\delta < 1$, it is obvious that for player i ($\{i\} = N(t)$), completing the provision immediately is the unique optimal action. Thus, $T(z) = t$. □

Step 2. Consider a subgame in period t such that $n(t) = \ell + 1$, $\sum_{i\in N(t)} V_i > R(t)$, and $t + \ell \leqslant \bar{T}$ for some $\ell \in \mathbb{N}$. Then, in each SPE of this subgame, $T(z) = t$.

Proof of Step 2. The existence of a SPE is shown as Part A. We can show that a strategy profile in which $\sum_{i\in N(t)} z_i(t) = R(t)$ and $z_i(t) \in (0, V_i)$ for each $i \in N(t)$ is a SPE in this subgame.

We now show that, in each SPE of this subgame, $T(z) = t$. The case for $\ell = 0$ is shown by Step 1. As an induction assumption, suppose that the statement of Step 2 is true for $\ell = k-1 \in \mathbb{N}$, and consider $\ell = k$.

The rest of the proof is divided into two claims.

Claim 1. *In each SPE, $T(z) < \infty$.*

Proof of Claim 1. To prove this claim, suppose, by contradiction, there is a SPE such that $T(z) = \infty$.

In this SPE, no one contributes in period t for the following reason. If a player makes a positive contribution in period t, since $\alpha < 1$, $\bar{T} < \infty$ and $T(z) = \infty$, the payoff must be negative. Then, no contribution in every period is a profitable deviation, which is a contradiction. Thus, for each $i \in N(t)$, $z_i(t) = 0$.

Next, we provide a profitable deviation. Let $j \in N(t)$. Consider a deviation by active player j so that $z_j(t) = \max\{R(t) - \sum_{i \in N(t) \setminus \{j\}} V_i, 0\} + \varepsilon < V_j$, where $\varepsilon > 0$ is a sufficiently small number. Moreover, as no other player contributes in period t, $N(t+1) = N(t) \setminus \{j\}$. Then, in the next period, $|N(t+1)| = k-1$ and $\sum_{i \in N(t+1)} V_i > R(t+1)$. Moreover, $t+1+k-1 = t+k \leqslant \bar{T}$. By the induction assumption, the provision is completed in period $t+1$. Therefore, player j's utility is $\delta^t(V_j - z_j(t)) > 0$. On the contrary, under the original contribution profile, player j's payoff is zero. Then, the deviation is profitable, which shows that the provision is completed by the deadline in a SPE. □

To complete the proof of Step 2, we show

Claim 2. *In each SPE, $T(z) = t$.*

Proof of Claim 2. To complete the proof, suppose by contradiction that there exists a SPE such that $T(z) > t$. Then, $n(t+1) > 0$. By Claim 1, the provision is completed by the deadline. Thus, the following conditions hold: $R(t+1) < \sum_{j \in N(t+1)} V_j$ and $\sum_{j \in N(t+1)} \sum_{t' \geqslant t+1} z_j(t') = R(t+1)$.

We have two cases.

Case 1. $n(t+1) = 1$. Let $\{j\} = N(t+1)$. In this case, if $z'_j(t) = z_j(T(z))$, the provision is completed in period t. As $\delta < 1$, it is a profitable deviation.

Case 2. $n(t+1) \geqslant 2$. Then, there exists a player, say, player $i \in N(t+1)$, such that, for some $t' \geqslant t+1$, $z_i(t') > 0$. Note that player i receives a surplus no greater than the total surplus of the active players in period $t+1$. That is, for sufficiently small $\varepsilon \in \mathbb{R}_{++}$,

$$\underbrace{V_i - z_i(t')}_{\text{player } i\text{'s surplus}} < \underbrace{\sum_{j \in N(t+1)} V_j - R(t+1)}_{\text{total surplus}} - \varepsilon. \tag{2}$$

In the SPE, player i's payoff is at most $\delta^t(V - z_i(t'))$.

Take a small ε satisfying (2) and $z_i(t') > \varepsilon > 0$. Consider i's deviation calling for $z'_i(t) = \max\{R(t+1)|_z - \sum_{j \in N(t+1)\setminus\{i\}} V_j, 0\} + \varepsilon > 0$.[14][15] Now, after the deviation, the new remaining amount is $R(t+1)|_{z_{-i}, z'_i} = R(t+1)|_z - z'_i(t) < \sum_{j \in N(t+1)\setminus\{i\}} V_j$. From the induction assumption, the provision is completed in period $t+1$. This is because the continuation game in period $t+1$ satisfies the induction assumption. Indeed, $|N(t+1)\setminus\{i\}| \leqslant k-1$ and $t+1+(k-1) = t+k \leqslant \bar{T}$. Then, player i's payoff is $\delta^t(V_i - [\max\{R(t+1) - \sum_{j \in N(t+1)\setminus\{i\}} V_j, 0\} + \varepsilon])$. From (2), $\varepsilon < z_i(t')$, and $t+1 \leqslant T(z)$, $\delta^t(V_i - [\max\{R(t+1) - \sum_{j \in N(t+1)\setminus\{i\}} V_j, 0\} + \varepsilon]) > \delta^t(V_i - z_i(t')) \geqslant \delta^{T(z)-1}(V_i - z_i(t'))$. The right-hand side is the payoff under the original strategy profile. Then, player i can make a profitable deviation.

In each case, a player can make a profitable deviation, which is a contradiction that the original contribution profile, z, is a SPE outcome. Then, it must be that $T(z) = t$. □

□

From Step 2, letting $\ell = n-1$ and $t = 1$ completes the proof. □

Part C: Proof of (c) We construct a SPE in which no contribution is made on the equilibrium path. To this end, consider the following strategy profile.

(i) For each $t \in \mathcal{T}$, if $R(t) = X^* - X(t-1) \geqslant \min\{\sum_{i \in N(t)} V_i, \sum_{k=1}^{\bar{T}+1-t} V^{(k)}_{N(t)}\}$, each player does not contribute. Here, $V^{(k)}_{N(t)}$ is the k-highest valuation among $N(t)$ players.

(ii) Otherwise, the active players complete the provision immediately.

Now we verify whether the strategy profile is a SPE. First, consider a deviation from (i). If $\sum_{i \in N(t)} V_i \leqslant \sum_{k=1}^{\bar{T}+1-t} V^{(k)}_{N(t)}$, the condition implies that $R(t) \geqslant \sum_{i \in N(t)} V_i$, in which case it is clear that no contribution is made in each SPE. Suppose that $\sum_{i \in N(t)} V_i > \sum_{k=1}^{\bar{T}+1-t} V^{(k)}$. Then, the condition implies that $R(t) \geqslant \sum_{k=1}^{\bar{T}+1-t} V^{(k)} \geqslant \max_{i \in N} V_i$. If $t = \bar{T}$, it is clear that no deviation is profitable. Now, consider $t < \bar{T}$. For a player's deviation to be profitable, it must hold that

[14] Here, $R(t)|_z$ is the remaining amount in period t under contribution profile z.

[15] Note also that, from (2), we can find that the new contribution amount $z'_i(t)$ is slightly smaller than the original contribution amount, $z_i(t')$.

$z_i(t) < V_i$. Since no other player contributes in period t, after i's deviation, we have the following inequality:

$$R(t+1) = R(t) - z_i(t) > \sum_{k=1}^{\bar{T}+1-t} V_{N(t)}^{(k)} - z_i(t) > \sum_{k=1}^{\bar{T}+1-t} V_{N(t)}^{(k)} - V_i \geqslant \sum_{k=1}^{\bar{T}+1-(t+1)} V_{N(t+1)}^{(k)}.$$

This inequality implies (i) for period $t+1$. Hence, no further contribution is made in the continuation play, and, thus, the provision is not completed. The deviating player wastes the contribution; therefore, this deviation is unprofitable. Thus, this strategy profile is a SPE. Since $R(1) = X^* > \sum_{k=1}^{\bar{T}} V_N^{(k)}$, no contribution is made on the equilibrium path.

From Part A, no deviation from (ii) is profitable. □

A.2. Proof of Proposition 1

From Theorem 1 (b), the provision is completed in period 1 in each SPE.

To conclude the proof, suppose, by contradiction, that $z_i(1) = 0$ for some $i \in N$. As the provision is completed in period 1, $R(2) = 0$. Then, this fact and by the assumption,

$$\sum_{k \in N \setminus \{i\}} z_k(1) \geqslant X^* > (1-\delta) \sum_{k \in N \setminus \{i\}} V_k.$$

This implies that at least one $j \neq i$, $z_j(1) > (1-\delta)V_j$.

Consider the deviation $z'_j(1) = \varepsilon > 0$ so that $V_i > z_j(1) - \varepsilon > (1-\delta)V_j$. Due to the assumption that $V_i > (1-\delta)V_j$, we can find such ε. Then, $R(2) = z_j(1) - \varepsilon < V_i$. After the deviation, from Theorem 1 (b), the provision is completed by players other than j in period 2, as $n - 1 \geqslant n(2) \geqslant 1$. Then, player j's payoff is $\delta V_j - \varepsilon$. From the definition of ε, the payoff is larger than the original payoff $V_j - z_j(1)$. Therefore, j's deviation is profitable. □

B. Formal analyses in Section 5.1

Section 5.1 (informally) discussed tightness of the assumptions made for Theorem 1 (b). The following subsections supplement the discussion with formal statements or examples.

B.1. Without the contribute-once restriction

As discussed in Section 5.1, without Assumption 1, as shown by Marx and Matthews (2000) (Proposition 5), the punishment strategy enables many contribution profiles, including inefficient ones, to be SPE outcomes.[16] The following fact provides the formal statement.

Fact 1 (Marx and Matthews, 2000)**.** *Suppose that $\alpha = 0$ and $V_i = V$ for each $i \in N$.*[17] *Without Assumption 1, the contribution profile z is a SPE outcome if and only if, for each $t \in \mathcal{T}$ and $i \in N$, $z_i(t) \in [0, V)$ and*

$$\delta^{T(z)-t}\left[V - \sum_{\tau=t}^{T(z)} z_i(\tau)\right] \geqslant \max\{0, V - [X^* - (X(t) - z_i(t))]\}. \tag{3}$$

This result shows that the efficient outcome (i.e., the public good is provided immediately) is a SPE outcome. On the contrary, no provision is also a SPE outcome, which is inefficient. Substituting $z_i(t) = 0$ for each $i \in N$ and $t \in \mathcal{T}$ satisfies (3). Less obviously, the following contribution profiles with delayed provision are also SPE outcomes. These outcomes are efficient if $\delta \to 1$. However, if not, they are inefficient, because of the delay in provision.

Example 1 (Delayed provision). Let $v = \frac{X^*}{n}$. Consider a profile such that, for each $i \in N$, $z_i(\bar{T}) = v$ and $z_i(t) = 0$ for each $t \neq \bar{T}$. Then, this profile satisfies (3). ▲

Example 2 (Gradualism). Suppose that $n > 2$, $\bar{T} \geqslant n - 1$, and $(n-1)V < X^* < nV$. Consider a profile such that, for each $i \in N \setminus \{n\}$, $z_i(i) = v$, $z_i(t) = 0$ for each $t \neq i$, $z_n(n-1) = v$, and $z_n(t) = 0$ for each $t \neq n - 1$. Then, this profile satisfies (3). ▲

In the last of this subsection, we prove this fact.

Proof of Fact 1. As the proof of Fact 1 is a variant of Marx and Matthews's (2000) Proposition 5, we only review a sketch here. The necessity is clear. The left-hand side of (3) is the continuation payoff of player i under contribution profile z. We can consider two types of deviations. One is not contributing after period t, which yields at least a zero payoff. The other is completing the provision immediately, which costs $X^* - (X(t) - z_i(t))$ for player $i \in N$. Then, player i's payoff is $V - [X^* - (X(t) - z_i(t))]$. For contribution profile z to be a SPE outcome, the yielding payoff cannot be exceeded by the payoffs attained by the above-mentioned deviations. This condition is summarized by in (3).

Under inequality (3), to implement contribution profile z, Marx and Matthews (2000) propose a strategy profile called the *grim-z strategy*. Under this strategy, on the equilibrium path, each player obeys contribution profile z. On any unilateral deviation, we have two cases. Let $z_i'(t)$ be the deviating contribution.

[16] As our framework slightly differs from that of Marx and Matthews (2000), we reformulate their Proposition 5 in our context.

[17] A similar result is obtained with asymmetric valuations; however, the condition becomes complex.

(i) $X^* - X(t) + z_i(t) - z'_i(t) < V$. Then, the deviating player, say player i, pays $X^* - (X(t) - z_i(t)) - z'_i(t)$, and the others pay nothing.

(ii) Otherwise, no player contributes after the deviation.

Then, player i's utility by this deviation is $\max\{0, V - X^* - (X(t) - z_i(t))\}$. Thus, this deviation is unprofitable under inequality (3). □

B.2. Equilibrium refinement

This subsection shows an example in which our main result no longer remains valid for a SPE allowing $z_i(t) = V_i$ for some $i \in N$ and $t \in \mathcal{T}$.

Example 3. Suppose that $N = \{1, 2\}$, $V_1 = V_2 = V$, and $X^* \in (V, 2V)$. Consider the following strategy profile: player 1 contributes $X^* - V$ in period 1, and player 2 contributes V in period 2. Clearly, player 1 has no incentive to deviate. In addition, player 2 also has no incentive to deviate, since the payoff is zero. Contributing in period 1 does not improve the payoff. Then, the strategy profile is a SPE in which the provision is completed in period 2. ▲

In this example, while there can be a SPE in which the provision is delayed, in no SPE is the provision delayed by more than two periods. To show this, suppose by contradiction that there is a SPE in which the provision is completed in period $t \geqslant 3$. Under such a strategy profile, a player (say, player 1) contributes $X^* - V$, and the other player (say, player 2) contributes in period $t \geqslant 3$. Then, player 1's utility is $\delta^{t-1}(2V - X^*)$. In this case, consider a deviation of player 1 such that he contributes $X^* - V + \varepsilon$ for some $\varepsilon > 0$ in period 1. Then, as shown in the proof of Theorem 1 (b), the provision is completed in period 2. Then, player 1's utility is $\delta(2V - X^* - \varepsilon)$. Since $\delta < 1$ and $t \geqslant 3$, if ε is sufficiently small, this deviation is profitable for player 1. This observation suggests that if the length of a period is sufficiently short, the delay is negligible. Indeed, in that case, even if we allow strategies in which $z_i(t) = V$ for some $i \in N$ and $t \in \mathcal{T}$, we can show that the provision is immediately completed in a SPE. See Appendix C for the formal analysis.

B.3. Withdrawal date

This subsection provides an example showing that, in a standard setup in which each contribution is borne in the committing period, when δ is small, the no-delay result fails. To see this, consider a two-period, symmetric two-player case. We also consider a contribution profile in which each player contributes in the second period. Suppose that each player contributes half of the needed burden, $X^*/2$. By making an earlier contribution with a smaller amount, the player exploits the other's surplus $V - X^*/2$. However, under the standard framework, as the earlier contribution is not discounted, it becomes a loss. Now, the deviating player's surplus is $\delta V - (X^* - V)$, while it is $\delta(V - X^*/2)$ in the original contribution profile. Then, the deviation is profitable if and only if $\delta(V - X^*/2) > (1 - \delta)(X^* - V)$. The left-hand side is the deviating player's exploiting surplus, while the right-hand side is the amount of the burden in the first period. If either the discount factor δ or the surplus $(V - X^*/2)$ is sufficiently small, this deviation is unprofitable and the provision is delayed.

B.4. Fully refundable case

In Section 5.1, we intuitively explain that when $\alpha = 1$, we can find a SPE in which the provision is not completed on the equilibrium path. The following proposition provides the formal statement.

Proposition 2. *Assume Assumption 1, $X^* > \max_{i \in N} V_i$, and $\alpha = 1$. Then, there is a SPE in which the provision is not completed by the deadline.*

Proof of Proposition 2. Consider the following profile such that $\sum_i z_i(1) < X^* - \max_{i \in N} V_i$ and $z_i(1) > 0$ for each $i \in N$. Then, the provision is not completed. Hence, for each $i \in N$, player i's payoff is zero. Consider a deviation of player $j \in N$, say, z'_j. If the deviation is profitable, the provision is completed and $V_j > \sum_{t \geqslant 1} z'_j(t)$. However, after period 1, since $z_i(1) > 0$ for each $i \in N \setminus \{j\}$, no other player can contribute any more. Then, the total contribution is less than X^*, and, therefore, the provision is not completed. Now, the payoff is also zero, and the deviation is unprofitable. □

B.5. Observability of the set of contributors

As discussed in Section 5.1, without the observability of the set of the contributors to date, we can find an equilibrium in which no contribution is made on the equilibrium path.

Example 4. Suppose that $N = \{1, 2, 3\}$, $\alpha = 0$, $V_i = V$ for each $i \in N$ and $X^* \in (2V, 3V)$. The setting is the common knowledge of the players. As the number of active players is unknown in some subgame, we employ the PBNE as the solution concept.

We rigorously define the strategy of the players. Let h_i be the history experienced by player i. Formally, h_i is an element of the set $\{(z_i(t), X(t))_{t \leqslant \tau} : \tau \leqslant \bar{T}\} \cup \{\varnothing\}$, where $X(t) = \sum_{\tau=1}^{t} \sum_{i \in N} z_i(\tau)$, and $\varnothing$ is the initial history. This is the set of sequences of player i's contribution $z_i(t)$ and the observed amount of total contribution $X(t)$. The strategy of player i, denoted by σ_i, is a function from the set of histories to a probability distribution over $[0, V)$. Player $i \in N$ has a belief over the other players' contribution $(z_j(t))_{j \neq i,t}$ conditioned on observed history. Let π_i be player i's belief, which is a function of histories to a probability distribution over $[0, V)^{N \setminus \{i\}}$.

We show a PBNE where no contribution is made. Consider the following strategy profile and off-the-equilibrium-path belief: each player makes no contribution if $R(t) \geqslant V$ in period t. Otherwise, they contribute $R(t)$ with probability 1. If player j observes a positive contribution amount in period t, that is, $X(t-1) > 0$, the player places a probability of 1 on the event that each $i \in N(t-1) \setminus \{j\}$ contributed in that period. We refer to the pair of this strategy and belief profile as (σ, π).

Let us check whether (σ, π) is a PBNE.[18] Suppose that $X(t) = 0$ (i.e., $R(t) = X^*$). Consider player i's deviation to contribute in period t. The amount should be less than V, and thus, $R(t+1) > V$. Then, as it is an off-the-equilibrium path, each player $j \in N \setminus \{i\}$ believes that only player j can contribute in period $t+1$ or later. As $R(t+1) > V$, player j makes no contribution, and the provision is never completed. Thus, player i wastes his contribution, which implies that this deviation is unprofitable.

Suppose $X(t) > 0$. Then, as it appears on an off-the-equilibrium-path, player j also places probability 1 on the event that $N(t) = \{j\}$. Thus, if $R(t) < V$, completing the provision is optimal; otherwise, making no contribution is optimal. Therefore, (σ, π) is a PBNE. Since $R(1) = X^* > 2V > V$, no contribution is made on the equilibrium path. ▲

C. Sufficiently short length of periods

This section provides a standard model given by Marx and Matthews (2000), in which the length of a period is sufficiently short. To this end, the set of periods is defined as $t \in \mathcal{T} = \underbrace{\{0, dt, 2dt, \ldots, \bar{T}\}}_{[\bar{T}/dt]+1 \text{ elements}}$ and each player $i \in N$'s payoff at time τ is defined as

$$u_i(z \mid \tau) := e^{-r(T(z)-\tau)} V_i - \sum_{t \in \mathcal{T}} e^{-r(t-\tau)} z_i(t), \quad r > 0.$$

Unlike our basic model in the main text, $\bar{T}$ can be infinite (i.e., $\bar{T} \in \mathbb{N} \cup \{\infty\}$), and the cost of contribution is borne at the committing period. As noted in Appendix B.2, we need not to assume that $z_i(t) < V_i$ for each $i \in N$ and $t \in \mathcal{T}$. In other words, any contribution amount is allowed. The following is a modified version of Theorem 1 (b), which shows that as $dt \to 0$,

[18] This profile is also a sequential equilibrium. See the working paper version of this study.

even without some of assumptions made for Theorem 1 (b), an approximate version of the no-delay result holds.

Theorem 2. *Suppose that Assumption 1 holds. Then, if* $X^* < \sum_{i\in N} V_i$ *and* $\bar{T} > 0$*, as* $dt \to 0$*,* $\sup\{T(z)\colon z \textit{ is a SPE profile}\} \to 0$.

Proof of Theorem 2. As in the proof of Theorem 1, the proof is an induction. Redefine $R(t) \coloneqq X^* - X(t-1)$ for each $t > 1$ and $R(0) \coloneqq X^*$.

Step 1. Consider a subgame in period t *such that* $n(t) = 1$*,* $i \in N(t)$*,* $R(t) < V_i$*, and* $t < \bar{T}$*. Then, in each SPE of this subgame,* $T(z) = t$*.*

Proof of Step 1. As this proof is obvious, it is omitted. □

Step 2. Consider a subgame in period $t < \bar{T}$ *such that* $n(t) = \ell + 1$ *and* $R(t) < \sum_{i\in N(t)} V_i$ *for some* $\ell \in \mathbb{N}$*. Then, as* $dt \to 0$*,* $\sup\{T(z) : z \textit{ is a SPE profile}\} \to t$*.*

Proof of Step 2. The case for $\ell = 0$ is implied from Step 1. As in the induction assumption, suppose that the statement is true if $\ell \leqslant k \in \mathbb{N}$. Now, consider $\ell = k + 1$.

Consider a subgame in period $t < \bar{T}$ such that $n(t) = k + 2$ and $R(t) < \sum_{i\in N(t)} V_i$.

As in the proof of Theorem 1(a), we can show that, in this subgame, a SPE exists.

In each SPE of this subgame, the completion period converges to $T(z) = t$. To show this, suppose, by contradiction, that there is a sequence of SPEs such that $\lim_{dt\to 0} T(z) > t$. Then, there is a SPE for arbitrary small dt, $n(t + t^*) > 0$, where $t^* = T(z) - t > 0$. From Step 1, we can show that $n(t + t^*) > 1$. This also implies that $n(t + dt) > 1$, since $t^* > dt$. As the public good is provided, $\sum_{i\in N(t+dt)} \sum_{t'\geqslant t+dt} z_i(t') = R(t + dt)$. We have the following two cases.

Case 1. Suppose that $R(t + dt) < \sum_{i\in N(t+dt)} V_i$. Let t_i be the contributing period of player i. Hence, consider player $i \in N(t+dt)$, such that $z_i(t_i) > 0$. As each player receives a non-negative payoff, for small $\varepsilon \in \mathbb{R}_{++}$,

$$V_i - z_i(t_i) < \sum_{j\in N(t+dt)} V_j - R(t + dt) - \varepsilon. \tag{4}$$

In the SPE, player i's payoff is at most $e^{-rt^*} V_i - e^{-rdt} z_i(t_i)$.

Take a small ε that satisfies (4) and $z_i(t_i) > \varepsilon > 0$. Consider i's deviation such that $z_i'(t) = \max\{R(t) - \sum_{j\in N(t+dt)\setminus\{i\}} V_j, 0\} + \varepsilon > 0$. Now, after the deviation, $R(t + dt) < \sum_{j\in N(t+dt)\setminus\{i\}} V_j$. From the induction assumption, as $dt \to 0$, the completion period is sufficiently close to $t + dt$. Then, player i's payoff is approximately $e^{-rdt} V_i - [\max\{R(t + 1) - \sum_{j\in N(t+dt)\setminus\{i\}} V_j, 0\} + \varepsilon])$. From (4), for sufficiently small dt and ε, this is a profitable deviation.

Case 2. Suppose that $R(t+dt) = \sum_{i \in N(t+dt)} V_i$. Then, $j \in N(t)$ such that $z_j(t) \in (0, V_j)$. Now, the payoff is $e^{-rt^*} V_j - z_j(t)$. Consider a deviation such that $z'_j(t) = z_j(t) + \varepsilon$, $\varepsilon > 0$. Then, after this deviation, $R(t+dt) < \sum_{i \in N(t+dt)} V_i$ for small $dt > 0$. From the induction assumption, the completion period converges to $t + dt$. Then, player j's payoff is $e^{-rdt} V_j - z_j(t) - \varepsilon$. For sufficiently small ε and dt, this deviation is profitable.

Both cases admit a profitable deviation, which contradicts to the supposition that z is a SPE outcome. □

By Step 2, letting $\ell = n - 1$ and $t = 0$ completes the proof. □

D. Formal analyses in Section 5.2

D.1. Linear public good

This subsection extends the basic model to the linear public good case given in the first paragraph of Section 5.2. For simplicity, focus on a two-player case.

Proposition 3. *Suppose that $\alpha = 0$, $n = \bar{T} = 2$, $\lambda_i = \lambda$ and $V_i = V$ for each $i \in N$. Assume that $X^* \in (V, 2V)$ and player i's utility is*

$$u_i(z) := \delta^{T(z)-1} \left(f_i(X(T(z))) - \sum_{t=1}^{\min\{T(z), \bar{T}\}-1} z_i(t) \right).$$

The definition of f_i is given in (1). In addition, suppose that Assumption 1 holds. Then, for sufficiently small $\lambda \geqslant 0$, in each SPE, the provision is completed in period 1.

Proof of Proposition 3. Suppose, by contradiction, that there is a SPE in which the provision is not completed in period 1 for each $\lambda > 0$. There is no SPE in which only player $i \in N$ contributes, as utility becomes $\delta(\lambda z_i - z_i) < 0$. We have the following four cases and show that each admits a profitable deviation.

Case 1. $z_1(1) > 0$, $z_2(1) > 0$, $z_1(1) + z_2(1) < X^*$, and $z_1(2) = z_2(2) = 0$. Consider player 1. If the provision is not completed, the utility is $\delta(\lambda(z_1(1) + z_2(1)) - z_1) = \delta(\lambda z_2(1) + (\lambda - 1) z_1(1))$. On the contrary, if the player does not contribute in any period, the payoff becomes $\delta \lambda z_2(1) > \delta(\lambda z_2(1) + (\lambda - 1) z_1(1))$ as $\lambda < 1$. Therefore, no SPE satisfies the conditions.

Case 2. $z_1(1) > 0$, $z_2(1) = z_1(2) = 0$, and $z_2(2) \geqslant 0$. In this case, if completing the provision is optimal for player 2, doing so in period 1 is a profitable deviation. If not completing is optimal for player 2, whose utility is $\delta(\lambda(z_1(1) + z_2(2)) - z_2(2)) \leqslant \delta \lambda z_1(1)$, then, for player 2, contributing nothing is optimal. Player 1's utility is $\delta\lambda z_1(1) - z_1(1) < 0$, and not contributing in all periods is a profitable deviation.

Case 3. $z_1(1) = z_2(1) = 0$, $z_1(2) > 0$, and $z_2(2) \geqslant 0$. If the provision is not completed in a SPE, player i's utility is $\delta(\lambda(z_1(2) + z_2(2)) - z_i(2))$. Then, not contributing is a profitable deviation.

Now, consider a SPE in which the provision is completed in period 1. Then, without loss of generality, we assume that $z_1(2) \geqslant z_2(2)$, which implies that $z_1(2) \geqslant X^*/2$. Now, consider the deviation such that

$$z_1'(1) = \frac{X^* - V}{1 - \lambda} + \varepsilon.$$

If λ and $\varepsilon > 0$ are sufficiently small, $z_1'(1) < X^*/2 \leqslant z_1(2)$. Moreover, after this deviation, for player 2, completing the provision is optimal. Now, player 1's utility is $\delta(V - z_1'(1)) > \delta(V - z_1(2))$. This shows that the deviation is profitable.

Case 4. $z_1(1) = z_2(1) = z_1(2) = z_2(2) = 0$. Consider the deviation that $z_1'(1) = \frac{X^*-V}{1-\lambda} + \varepsilon$. Then, for sufficiently small λ and small $\varepsilon > 0$, $V - z_1'(1) > 0$, and, in period 2, player 2 completes the provision. Then, it is a profitable deviation.

Finally, we verify that completing the provision in period 1 is a SPE. To see this, let us consider the contribution profile $z_1(1) = z_2(1) = X^*/2$. Making a greater contribution is unprofitable. Consider a lower contribution by player i, say $z_i'(1)$. Then, player i's utility becomes $\delta(\lambda X^*/2 + (\lambda - 1)z_i'(1))$. Therefore, such a deviation is unprofitable if and only if $V - X^*/2 > \delta(\lambda X^*/2 + (\lambda - 1)z_i'(1))$. If λ is sufficiently small, for any $z_i'(1) \geqslant 0$, this inequality holds. □

D.2. Incomplete information

This section studies an extension to the incomplete information. For tractability, assume a two-player, two-period case and $\alpha = 0$. Each player's valuation V_i is randomly drawn from $\{V_L, V_H\}$ with probability $p = \Pr(V_i = V_L) \in (0, 1)$. Further, assume that $V_L < V_H < X^* < 2V_L$. This assumption implies that, for each valuation profile, completing the provision is optimal. The first proposition states that each player necessarily contributes a positive amount in period 1 in each PBNE.

Proposition 4. *Assume the above settings and Assumption 1. Then, in any pure-strategy, type-symmetric PBNE, each type of player contributes in period* 1.

Proof. See Appendix D.2.1. □

The above proposition does not guarantee the completion of the provision: even if each player makes a positive contribution at period 1, the total contribution may fall short of the required amount, due to the uncertainty of the contribution amount tied to the uncertainty of the preferences. The following proposition characterizes the PBNE.

Proposition 5. *Suppose that all the assumptions of Proposition 4 hold. Then, the possible pure-strategy, type-symmetric PBNEs are the following.*

Type 1: The provision is completed in period 1 *with a probability of* 1.

Type 2: The provision is completed in period 1 *with a probability of* $1 - p^2$ *and never completed with the complementary probability. This type of equilibrium exists if and only if a* z_H *satisfies the following inequalities:*

$$V_H \geqslant z_H \geqslant X^* - (1-p)V_L \tag{5}$$

$$p\frac{V_H}{2} + \frac{X^*}{2} \geqslant z_H \geqslant p\frac{V_L}{2} + \frac{X^*}{2} \tag{6}$$

$$\frac{(1-\delta)V_H + \delta(1-p)X^*}{1-\delta(2p-1)} \geqslant z_H \geqslant \frac{-(1-p-\delta)V_L + (1-\delta(1-p))X^*}{(1+\delta(2p-1))}. \tag{7}$$

Proof. See Appendix D.2.2. □

In a type 1 equilibrium, each player pays $X^*/2$ in period 1, and the outcome is efficient. On the other hand, type 2 equilibrium is inefficient. In this equilibrium, the high type bears the greater burden. As the low type expects the high type's greater contribution, he contributes less. Then, if both of the players are low type, since no more contribution can made, the provision fails. To ensure that such a contribution profile be an equilibrium, δ should be small. Otherwise, since the cost of delayed provision is negligible, a low type player would deviate to wait in period 1 to complete the provision. Indeed, substituting $\delta = 1$ into (7) yields

$$\frac{1}{2}X^* \geqslant z_H \geqslant \frac{1}{2}(V_L + X^*).$$

Then, no z_H satisfies (7), and a type 2 outcome is unachievable. Therefore, type 1 is the unique PBNE outcome. If the length of the period is sufficiently short, it approximates the case for $\delta \approx 1$. This suggests that, in any SPE of a continuous time model, the provision is completed immediately and surely.

D.2.1. Proof of Proposition 4

To complete the proof, consider contribution profiles such that some type of player does not contribute in period 1. We show that each admits a profitable deviation.

Case 1. Consider a strategy such that each player does not contribute in any period, irrespective of the type of player. On the equilibrium path, each player earns nothing. Then, as in the case of symmetric information, player $i \in N$ has the incentive to deviate so that $z_i(1) = X^* - V_L + \varepsilon < V_L$, which incentivizes the other player to complete the provision in period 2. Therefore, if $\varepsilon > 0$ is sufficiently small, such a deviation is profitable.

Case 2. Consider a strategy such that, irrespective of the type of player, each player does not contribute in period 1 and the type $\theta \in \{L, H\}$ player contributes z_θ in period 2. As this is a PBNE and the type θ player has an incentive to contribute, the provision is completed with a probability. Therefore, for at least one θ, $z_\theta \geqslant X^*/2$. Otherwise, the provision is never completed. Without loss of generality, let $\theta = H$. Under this strategy, the provision is completed with a probability of less than or equal to 1.

On the contrary, for a type H player, by deviating to contribute $X^* - V_L + \varepsilon$, the other player is incentivized to complete the provision, irrespective of type. Then, the provision is completed with a probability of 1, and the deviating player's burden decreases. This is a profitable deviation for the type H player.

Case 3. Consider a strategy such that the type θ player contributes in period 1 and the type $\theta' \neq \theta$ player does not contribute in period 1.

Without loss of generality, assume that $\theta = H$ and $\theta' = L$. Let z_H be the type H player's contribution amount in period 1. We have the following two subcases.

Subcase 1. Consider a strategy such that if no contribution is made in period 1, each player does not contribute.

For the type H player, as contributing at period 1 is optimal, $z_H \in (X^* - V_L, X^*/2]$. Otherwise, as $X^* - V_L < X^*/2$, the provision is never completed.

Under this strategy, the type L player's expected utility is $p \cdot 0 + (1-p)\delta(V_L - (X^* - z_H))$. Now, consider the type L player's deviation such that he contributes $X^* - z_H < V_L$ in period 1. If the other player's type is H, the provision is completed in period 1. If the other player is type L, the contribution is $z_H < X^*/2 < V_L$ in period 2. Therefore, the deviating player's expected utility is $p\delta(V_L - (X^* - z_H)) + (1-p)(V_L - (X^* - z_H))$, which is larger than the expected utility under the original strategy profile. Thus, we find a profitable deviation.

Subcase 2. Consider a strategy such that if no contribution is made in period 1, the players complete the provision in period 2. In this case, $z_L = X^*/2$.

As before, we have $z_H \in (X^* - V_L, X^*/2]$. Hence, the type H player's expected utility is

$$u_H(z_H) = \begin{cases} -\delta z_H & \text{if} \quad z_H \leqslant X^* - V_L, \\ p\delta(V_H - z_H) & \text{if} \quad z_H \in (X^* - V_L, X^*/2) \\ p(V_H - X^*/2) + (1-p)\delta(V_H - X^*/2) & \text{if} \quad z_H = X^*/2. \end{cases}$$

Note that $u_H(z_H)$ is discontinuous at $z_H = X^* - V_L$ because as each player takes a strategy such that $z_i < V_i$, if the type L player observes $z_H = X^* - V_L$, the player contributes nothing. Therefore, $z_H < X^*/2$ is never optimal. Then, as we consider a PBNE such that $z_H > 0$, $z_H = X^*/2$.

Then, the type L player's expected utility is $p\delta(V_L - X^*/2) + (1-p)\delta(V_L - X^*/2)$. Now, consider the type L player's deviation to contribute $X^*/2$ in period 1. The expected utility is $p\delta(V_L - X^*/2) + (1-p)(V_L - X^*/2)$, which is an improvement. The above discussion shows that, in any PBNE, each type of player contributes in period 1. □

D.2.2. Proof of Proposition 5

From Proposition 4, we focus on the strategy profile, in which each type of player contributes in period 1. For each $\theta \in \{L, H\}$, let $z_\theta > 0$ be the type θ player's contribution amount in period 1.

Consider $z_H \geqslant z_L > 0$. We have the following cases.

Case 1. $2z_H < X^*$. In this case, the provision is never completed, and, thus, each player wastes their contribution. Then, not contributing is a profitable deviation.

Case 2. $z_H + z_L < X^* \leqslant 2z_H$. From the viewpoint of the type L player, the provision is never completed, and, thus, the type L player wastes the contribution. Therefore, not contributing is a profitable deviation.

Case 3. $2z_L < X^* \leqslant z_H + z_L$. Under this strategy profile, from Assumption 1, the provision is completed if and only if one of the players is type H. Therefore, the provision is completed in period 1 with a probability of $1 - p^2$. For this profile to be a PBNE outcome, $z_H + z_L = X^*$.

To specify the condition for this type of strategy to be a PBNE, first consider the type H player's incentive. Under the strategy profile, this player's expected utility is $V_H - z_H$. We can consider two types of deviations. The first is contributing $z'_H = z_L = X^* - z_H$ in period 1. Under this deviation, if the other player's type is L, the provision fails. On the contrary, if the other player's type is H, and as the player contributes z_H, the provision is completed in period 1. Therefore, the expected utility from this deviation is $(1-p)V_H - z_L$. The second deviation is delaying the contribution. If the other player's type is L, the deviating player completes the provision by contributing $z'_H = z_H$ in period 2. If the other player's type is H, the deviating player completes the provision by contributing $z'_H = z_L$ in period 2. Then, the expected utility of this strategy is $\delta[V_H - (pz_H + (1-p)z_L)]$. To summarize, for such a contribution profile to be a PBNE outcome, the following inequalities hold:

$$V_H - z_H \geqslant 0, \tag{8}$$

$$V_H - z_H \geqslant (1-p)V_H - z_L, \tag{9}$$

$$V_H - z_H \geqslant \delta V_H - \delta(pz_H + (1-p)z_L). \tag{10}$$

For the type L player, a similar argument holds. Under the original strategy profile, if both players' types are L, the provision is never completed. Then, the following inequalities are needed:

$$(1-p)V_L - z_L \geqslant 0, \tag{11}$$

$$(1-p)V_L - z_L \geqslant V_L - z_H, \tag{12}$$

$$(1-p)V_L - z_L \geqslant \delta[V_L - pz_H - (1-p)z_L]. \tag{13}$$

Hence, the strategy profile such that the type θ player contributes z_θ is a PBNE if and only if inequalities (8)–(13) hold. Inequality (12) is rearranged to $z_H - z_L \geqslant pV_L$. Moreover, from inequalities (8) and (11), and $z_H + z_L = X^*$,

$$V_H \geqslant z_H \geqslant X^* - (1-p)V_L. \tag{14}$$

From inequalities (9) and (12),

$$p\frac{V_H}{2} + \frac{X^*}{2} \geqslant z_H \geqslant p\frac{V_L}{2} + \frac{X^*}{2}. \tag{15}$$

From inequalities (10) and (13), we also have

$$\frac{(1-\delta)V_H + \delta(1-p)X^*}{1-\delta(2p-1)} \geqslant z_H \geqslant \frac{-(1-p-\delta)V_L + (1-\delta(1-p))X^*}{(1+\delta(2p-1))}. \tag{16}$$

These inequalities are those shown in the statement of the proposition. This is the type 2 equilibrium.

Case 4. $2z_L \geqslant X^*$. In this case, it must be that $z_L = z_H = X^*/2$ is the unique equilibrium outcome. One can show that this strategy profile is a PBNE by the same way in the proof of Theorem 1(a), in which case, the provision is completed with a probability of 1 (type 1 equilibrium).

Replacing the roles of H and L for $z_L > z_H > 0$, as in cases 1 and 2, if $z_H + z_L < X^*$, it admits a contradiction. As in case 3, if $2z_H < X^*$, the following inequality is needed:

$$(1-p)\frac{V_L}{2} + \frac{X^*}{2} \geqslant z_L \geqslant (1-p)\frac{V_H}{2} + \frac{X^*}{2}.$$

However, as $V_H > V_L$, no z_L satisfies the above inequality. Thus, in this case, there is no type-symmetric PBNE. □